

# Contents

*Chapters are divided into sections. Some topics occupy several sections. We give a name to a section and list it here only when it starts a new topic.*

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