# Contents


Preface . . . . . vii
Introduction . . . . . xi

**1 Background** **1**
1.1 Valuations . . . . . 1
1.2 Completions . . . . . 16
1.3 Differential Forms . . . . . 24
1.4 Residues . . . . . 30
1.5 Exercises . . . . . 37

**2 Function Fields** **40**
2.1 Divisors and Adeles . . . . . 40
2.2 Weil Differentials . . . . . 47
2.3 Elliptic Functions . . . . . 52
2.4 Geometric Function Fields . . . . . 54
2.5 Residues and Duality . . . . . 58
2.6 Exercises . . . . . 64

**3 Finite Extensions** **68**
3.1 Norm and Conorm . . . . . 69
3.2 Scalar Extensions . . . . . 72
3.3 The Different . . . . . 75
3.4 Singular Prime Divisors . . . . . 82
3.5 Galois Extensions . . . . . 89
3.6 Hyperelliptic Functions . . . . . 93

3.7 Exercises . . . . . 99

**4 Projective Curves** **103**
4.1 Projective Varieties . . . . . 103
4.2 Maps to $\mathbb{P}^n$ . . . . . 108
4.3 Projective Embeddings . . . . . 114
4.4 Weierstrass Points . . . . . 122
4.5 Plane Curves . . . . . 136
4.6 Exercises . . . . . 147

**5 Zeta Functions** **150**
5.1 The Euler Product . . . . . 151
5.2 The Functional Equation . . . . . 154
5.3 The Riemann Hypothesis . . . . . 156
5.4 Exercises . . . . . 161

**A Elementary Field Theory** **164**

**References** **175**

**Index** **177**