

CONTENTS

Notes to the Reader	vii
Preface	xiii
CHAPTER I. SETS AND MAPPINGS	1
1. The Axioms of Set Theory	1
2. Correspondences	9
3. Mappings and Quotient Sets	12
4. Ordered Sets	18
5. Cardinals and Ordinals	28
6. Categories and Functors	36
CHAPTER II. ALGEBRAIC STRUCTURES	41
1. Closure Systems	41
2. Ω -Algebras	47
3. The Isomorphism Theorems	57
4. Lattices	63
5. The Lattice of Subalgebras	79

6. The Lattice of Congruences	86
7. Local and Residual Properties	99
8. The Lattice of Categories of Ω -Algebras	104
CHAPTER III. FREE ALGEBRAS	108
1. Universal Functors	108
2. Ω -Word Algebras	116
3. Clones of Operations	126
4. Representations in Categories of Ω -Algebras	132
5. Free Algebras in Categories of Ω -Algebras	137
6. Free and Direct Composition of Ω -Algebras	142
7. Derived Operators	145
8. Presentations of Ω -Algebras	150
9. The Word Problem	154
CHAPTER IV. VARIETIES	161
1. Definition and Basic Properties	161
2. Free Groups and Free Rings	165
3. The Generation of Varieties	169
4. Representations in Varieties of Algebras	180
CHAPTER V. RELATIONAL STRUCTURES AND MODELS	188
1. Relational Structures over a Predicate Domain	188
2. Boolean Algebras	191
3. Derived Predicates	200
4. Closed Sentence Classes and Axiomatic Model Classes	205
5. Ultraproducts and the Compactness Theorem	209
6. The Model Space	213

CHAPTER VI. AXIOMATIC MODEL CLASSES	220
1. Reducts and Enlargements	220
2. The Local Determination of Classes	222
3. Elementary Extensions	228
4. \mathcal{P} -Closed Classes and Quasivarieties	233
5. Classes Admitting Homomorphic Images	236
6. The Characterization of Axiomatic Model Classes	239
CHAPTER VII. APPLICATIONS	247
1. The Natural Numbers	247
2. Abstract Dependence Relations	252
3. The Division Problem for Semigroups and Rings	263
4. The Division Problem for Groupoids	279
5. Linear Algebras	283
6. Lie Algebras	289
7. Jordan Algebras	297
BIBLIOGRAPHY	310
LIST OF SPECIAL SYMBOLS	319
INDEX OF NAMES	325
INDEX OF SUBJECTS	327