

Contents

| | |
|--|-----------|
| Preface | ix |
| 1 C^*-Algebra Theory | 1 |
| 1.1 C^* -algebras and $*$ -homomorphisms | 1 |
| 1.2 Spectral theory | 5 |
| 1.3 Matrix algebras | 9 |
| 1.4 Exercises | 10 |
| 2 Projections and Unitary Elements | 15 |
| 2.1 Homotopy classes of unitary elements | 15 |
| 2.2 Equivalence of projections | 21 |
| 2.3 Semigroups of projections | 28 |
| 2.4 Exercises | 30 |
| 3 The K_0-Group of a Unital C^*-Algebra | 35 |
| 3.1 Definition of the K_0 -group of a unital C^* -algebra | 35 |
| 3.2 Functoriality of K_0 | 41 |
| 3.3 Examples | 46 |
| 3.4 Exercises | 54 |
| 4 The Functor K_0 | 59 |
| 4.1 Definition and functoriality of K_0 | 59 |
| 4.2 The standard picture of the group $K_0(A)$ | 62 |
| 4.3 Half and split exactness and stability of K_0 | 66 |
| 4.4 Exercises | 70 |
| 5 The Ordered Abelian Group $K_0(A)$ | 77 |
| 5.1 The ordered K_0 -group of stably finite C^* -algebras | 77 |
| 5.2* States on $K_0(A)$ and traces on A | 82 |
| 5.3 Exercises | 83 |

| | | |
|-----------|--|------------|
| 6 | Inductive Limit C^*-Algebras | 89 |
| 6.1 | Products and sums of C^* -algebras | 89 |
| 6.2 | Inductive limits | 91 |
| 6.3 | Continuity of K_0 | 97 |
| 6.4* | Stabilized C^* -algebras | 101 |
| 6.5 | Exercises | 104 |
| 7 | Classification of AF-Algebras | 109 |
| 7.1 | Finite dimensional C^* -algebras | 109 |
| 7.2 | AF-algebras | 113 |
| 7.3 | Elliott's classification theorem | 118 |
| 7.4* | UHF-algebras | 125 |
| 7.5 | Exercises | 130 |
| 8 | The Functor K_1 | 133 |
| 8.1 | Definition of the K_1 -group | 133 |
| 8.2 | Functoriality of K_1 | 138 |
| 8.3* | K_1 -groups and determinants | 144 |
| 8.4 | Exercises | 147 |
| 9 | The Index Map | 153 |
| 9.1 | Definition of the index map | 154 |
| 9.2 | The index map and partial isometries | 158 |
| 9.3 | An exact sequence of K -groups | 163 |
| 9.4* | Fredholm operators and Fredholm index | 165 |
| 9.5 | Exercises | 169 |
| 10 | The Higher K-Functors | 175 |
| 10.1 | The isomorphism between $K_1(A)$ and $K_0(SA)$ | 175 |
| 10.2 | The long exact sequence in K -theory | 178 |
| 10.3 | Exercises | 182 |
| 11 | Bott Periodicity | 185 |
| 11.1 | The Bott map | 185 |
| 11.2 | The proof of Bott periodicity | 187 |
| 11.3 | Applications of Bott periodicity | 199 |
| 11.4* | Homotopy groups and K -theory | 201 |
| 11.5* | The holomorphic function calculus | 204 |
| 11.6 | Exercises | 206 |

| | |
|--|------------|
| 12 The Six-Term Exact Sequence | 209 |
| 12.1 The exponential map and the six-term exact sequence | 209 |
| 12.2 An explicit description of the exponential map | 211 |
| 12.3 Exercises | 215 |
| 13 Inductive Limits of Dimension Drop Algebras | 219 |
| 13.1 Dimension drop algebras | 219 |
| 13.2 Countable Abelian groups as K -groups | 222 |
| 13.3 Exercises | 228 |
| References | 231 |
| Table of K-groups | 234 |
| Index of symbols | 236 |
| General index | 239 |