

TABLE OF CONTENTS

<i>Preface</i>	5
<i>Table of contents</i>	9
<i>Table of notations</i>	12
CHAPTER I. Basic concepts. The most important groups	
§ 1. Notation and terminology	13
§ 2. Direct sums	17
§ 3. Cyclic groups	22
§ 4. Quasicyclic groups	23
§ 5. The additive group of the rationals	25
§ 6. The p -adic integers	26
§ 7. Operator modules	27
§ 8. Linear independence and rank	29
Exercises	34
CHAPTER II. Direct sum of cyclic groups	
§ 9. Free (abelian) groups	37
§ 10. Finite and finitely generated groups	39
§ 11. Direct sums of cyclic p -groups	43
§ 12. Subgroups of direct sums of cyclic groups	45
§ 13. Two dual criteria for the basis	47
§ 14. Further criteria for the existence of a basis	50
Exercises	52
CHAPTER III. Divisible groups	
§ 15. Divisibility by integers in groups	57
§ 16. Homomorphisms into divisible groups	59
§ 17. Systems of linear equations over divisible groups	60
§ 18. The direct summand property of divisible groups	62
§ 19. The structure theorem on divisible groups	64
§ 20. Embedding in divisible groups	65
Exercises	67
CHAPTER IV. Direct summands and pure subgroups	
§ 21. Direct summands	71
§ 22. Absolute direct summands	73
§ 23. Pure subgroups	76
§ 24. Bounded pure subgroups	79
§ 25. Factor groups with respect to pure subgroups	81
§ 26. Algebraically compact groups	83
§ 27. Generalized pure subgroups	87
§ 28. Neat subgroups	91
Exercises	93

CHAPTER V. *Basic subgroups*

§ 29. Existence of basic subgroups. The quasibasis	97
§ 30. Properties of basic subgroups	101
§ 31. Different basic subgroups of a group	103
§ 32. The basic subgroup as an endomorphic image Exercises	106 108

CHAPTER VI. *The structure of p -groups*

§ 33. p -groups without elements of infinite height	111
§ 34. Closed p -groups	114
§ 35. The Ulm sequence	117
§ 36. ZIPPIN'S theorem	121
§ 37. ULM'S theorem	123
§ 38. Construction of groups with a prescribed Ulm sequence	127
§ 39. Non-isomorphic groups with the same Ulm sequence	134
§ 40. Some applications	135
§ 41. Direct decompositions of p -groups Exercises	137 141

CHAPTER VII. *Torsion free groups*

§ 42. The type of elements. Groups of rank 1	145
§ 43. Indecomposable groups	150
§ 44. Torsion free groups over the p -adic integers	154
§ 45. Countable torsion free groups	157
§ 46. Completely decomposable groups	162
§ 47. Complete direct sums of infinite cyclic groups. Slender groups	168
§ 48. Homogeneous groups	173
§ 49. Separable groups Exercises	176 179

CHAPTER VIII. *Mixed groups*

§ 50. Splitting mixed groups	185
§ 51. Factor groups of free groups	192
§ 52. A characterization of arbitrary groups by matrices	196
§ 53. Groups over the p -adic integers Exercises	198 200

CHAPTER IX. *Homomorphism groups and endomorphism rings*

§ 54. Homomorphism groups	205
§ 55. Endomorphism rings	210
§ 56. The endomorphism ring of p -groups	214
§ 57. Endomorphism rings with special properties	218
§ 58. Automorphism groups	221
§ 59. Fully invariant subgroups Exercises	224 227

CHAPTER X. *Group extensions*

§ 60. Extensions of groups	233
§ 61. The group of extensions	236
§ 62. Induced endomorphisms of the group of extensions	239
§ 63. Structural properties of the group of extensions Exercises	243 247

CHAPTER XI. *Tensor products*

§ 64. The tensor product	249
§ 65. The structure of tensor products	254
Exercises	256

CHAPTER XII. *The additive group of rings*

§ 66. Ideals determined by the additive group	259
§ 67. Multiplications on a group	261
§ 68. Rings on direct sums of cyclic groups	263
§ 69. Torsion rings	265
§ 70. Torsion free rings	268
§ 71. Nil groups and quasi nil groups	272
§ 72. The additive group of Artinian rings	280
§ 73. Artinian rings without subgroups of type p^∞	283
§ 74. The additive group of semi-simple and regular rings	286
§ 75. The additive group of rings with maximum or restricted minimum condition	288
Exercises	290

CHAPTER XIII. *The multiplicative group of fields*

§ 76. Finite algebraic extensions of prime fields	295
§ 77. Algebraically and real closed fields	297
Exercises	298

CHAPTER XIV. *The lattice of subgroups*

§ 78. Properties of the subgroup lattice	300
§ 79. Projectivities. Projectivities of cyclic groups	303
§ 80. Projectivities of torsion groups	305
§ 81. Projectivities of torsion free and mixed groups	309
§ 82. Dualisms	311
Exercises	312

CHAPTER XV. *Decompositions into direct sums of subsets*

§ 83. Decompositions of cyclic groups	315
§ 84. Decompositions into weakly periodic subsets	318
§ 85. Decompositions into an infinity of components	324
Exercises	329

CHAPTER XVI. *Various questions*

§ 86. Hereditarily generating systems	332
§ 87. Universal homomorphic images	336
§ 88. Universal subgroups	341
§ 89. A combinatorial problem	344
Exercises	350

<i>Bibliography</i>	353
<i>Author index</i>	363
<i>Subject index</i>	365