

Contents

Introduction	1
Acknowledgements	4
1. Notation and Background	5
1.1. Generalities	5
1.2. Rational Groups	6
1.3. The Anatomy of Finite Posets	13
1.4. Unit Groups	17
1.5. Integral Linear Algebra	19
1.6. Exercises	23
1.7. Solutions to Exercises	25
2. Basics and Completely Decomposable Groups	27
2.1. Rank, Coefficient Groups, and Purity	27
2.2. Rank-One Groups and Types	30
2.3. Socles and Radicals	32
2.4. Completely Decomposable Groups	36
2.5. Bounded Quotients of Completely Decomposable Groups	43
2.6. Homocyclic Finite Groups	49
2.7. Exercises	51
2.8. Questions	57
2.9. Solutions to Exercises	57
3. Cyclic Essential Extensions	63
3.1. Cyclic Essential Extensions	63
3.2. Examples	65
3.3. Exercises	70
4. Regulating Subgroups and Regulators	73
4.1. Butler Groups	74

4.2.	Regulating Subgroups	78
4.3.	Counting Regulating Subgroups	85
4.4.	The Regulator	87
4.5.	Odds and Ends	93
4.6.	Exercises	98
4.7.	Questions	100
4.8.	Solutions to Exercises	101
5.	Local-Global Relationships	103
5.1.	Primary Constituents	103
5.2.	Localization of the Regulator Criterion	106
5.3.	Regulating Subgroups of Primary Constituents	107
5.4.	Questions	108
6.	Groups with Cyclic Regulating Quotient	109
6.1.	Notation and Basics	110
6.2.	Regulating Subgroups	111
6.3.	Clipped crq-Groups	114
6.4.	Local crq-Groups	118
6.5.	Direct Decomposition of Local crq-Groups	119
6.6.	Regulating Quotients of Local crq-Groups	127
6.7.	Exercises	133
6.8.	Questions	136
6.9.	Solutions to Exercises	136
7.	Completely Decomposable Summands	141
7.1.	Width and Rank	141
7.2.	Radicals and Co-Butler Decompositions	143
7.3.	Completely Decomposable Summands and Clipped Groups	146
7.4.	Exercises	151
7.5.	Questions	152
7.6.	Solutions to Exercises	153
8.	Anti-Representations	155
8.1.	Anti-Representations	156
8.2.	Isomorphism	161
8.3.	Exercises	169
8.4.	Solutions to Exercises	170
9.	Near-Isomorphism and Type-Isomorphism	171
9.1	Near-Isomorphism	171

9.2.	Type-Isomorphism is Near-Isomorphism	176
9.3.	Exercises	181
9.4.	Questions	181
9.5.	Solutions to Exercises	182
10.	Fundamental Decomposition Theorems	185
10.1.	Lifting Idempotents	186
10.2.	Arnold's Theorem	189
10.3.	The Azumaya Theorem	192
10.4.	The Faticoni–Schultz Theorem	195
10.5.	Exercises	197
10.6.	Questions	198
10.7.	Solutions to Exercises	198
11.	Finite Essential Extensions	199
11.1.	Standard Description	199
11.2.	Greatest Common Divisors	202
11.3.	Computation of Greatest Common Divisors	208
11.4.	Purification of Base Group Summands	210
11.5.	Burkhardt Invariants	212
11.6.	Primary Constituents	214
11.7.	Representing Matrices	215
11.8.	Two Existence Theorems	216
11.9.	Completely Decomposable Direct Summands	218
11.10.	A Big Example and Maple Procedures	221
11.11.	Some Additional Maple Procedures	237
11.12.	Exercises	238
11.13.	Questions	242
11.14.	Solutions to Exercises	242
12.	Classification	249
12.1.	crq-Group	249
12.2.	Near-Isomorphism Invariants	252
12.3.	Two Critical Types	254
12.4.	Homocyclic Regulator Quotients	257
12.5.	Uniform Groups	259
12.6.	Global Block-Rigid crq-Groups and Isomorphism	264
12.7.	Some Applications	269
12.8.	Exercises	272
12.9.	Questions	276
12.10.	Solutions to Exercises	276

13. Direct Decompositions of Block-Rigid crq-Groups	281
13.1. Decompositions	281
13.2. Exercises	292
13.3. Solutions to Exercises	292
14. In Search for Good Categories	293
14.1. Some Natural Categories	293
14.2. Categories of ACD Groups and Anti-Representations	294
14.3. Exercises	299
14.4. Questions	301
14.5. Solutions to Exercises	301
15. Associated Structures	303
15.1. Endomorphism Rings of Completely Decomposable Groups	303
15.2. Endomorphism Rings of Almost Completely Decomposable Groups	306
15.3. Endomorphism Rings of Rigid crq-Groups	307
15.4. Endomorphism Rings and Type Endomorphism Rings	312
15.5. The Structure of $\text{TypEnd}_x \bar{A}$	313
15.6. Exercises	314
15.7. Questions	316
15.8. Solutions to Exercises	317
16. Literature	323
16.1. Regulating Hulls	323
16.2. Tight Subgroups	325
16.3. Pure Subgroups and Torsion-Free Quotients	326
16.4. High and Low Covers	327
16.5. The Invariants of Vinsonhaler	330
16.6. The Ladder Phenomenon	332
16.7. Further Papers	333
16.8. Exercises	334
16.9. Questions	337
16.10. Solutions to Exercises	338
Bibliography	343
Index	347