

# Contents

Preface . . . . .	vii
Notation and Terminology . . . . .	xiii

## Part A: Finite Simple Groups and Their Classification

1. Introduction: A Survey of the Classification Project by <i>Michael J. Collins</i>	
1.1 Background . . . . .	3
1.2 Gorenstein's Programme . . . . .	9
1.3 The Known Simple Groups . . . . .	11
1.4 Nonconnected Groups. . . . .	26
1.5 Groups of Component Type . . . . .	30
1.6 Groups of Noncomponent Type . . . . .	36
2. Standard Subgroups in Finite Groups by <i>Gary M. Seitz</i>	
2.1 Introduction . . . . .	41
2.2 Reduction to the Case of $R$ Cyclic . . . . .	44
2.3 $\tilde{A} \cong G(2^a)$ . . . . .	47
2.4 $\tilde{A} \cong A_n, n \geq 5$ . . . . .	53
2.5 $\tilde{A} \cong G(q), q$ odd . . . . .	56
2.6 Sporadic Groups . . . . .	62
3. The $B(G)$ -Conjecture and Unbalanced Groups by <i>Ronald Solomon</i>	
3.1 A Brief Prehistory . . . . .	63
3.2 Signalizer Functors . . . . .	68
3.3 The Unbalanced Group Conjecture . . . . .	73
3.4 Aschbacher's Theorem and its Consequences . . . . .	74
3.5 The Alternating Groups . . . . .	76
3.6 Pumping up Unbalancing Components . . . . .	78
3.7 The $A_7$ and $L_2(q)$ Problems, Phase I . . . . .	81
3.8 The $L_3(4)$ and $He$ Cases . . . . .	82
3.9 The $L_2(q)$ Problem, Phase II . . . . .	84
3.10 Other Approaches . . . . .	85
4. Finite Groups of Characteristic 2 Type by <i>Daniel Gorenstein and Richard Lyons</i>	
4.1 Introduction . . . . .	89

4.2	Definitions and Statements of the Main Results . . . . .	90
4.3	General Results and Properties of $\mathcal{K}$ -groups . . . . .	96
4.4	Theorem A . . . . .	103
4.5	Theorem B . . . . .	112
4.6	Theorem C . . . . .	122
5.	The Uniqueness Case for Groups of Characteristic 2 Type by <i>Michael Aschbacher</i>	
5.1	Introduction . . . . .	133
5.2	The Characteristic Core . . . . .	134
5.3	Weak Closures . . . . .	139
5.4	$TI$ -Sets . . . . .	141
5.5	$GF(2)$ -Representations. . . . .	143
5.6	2-Reduced Weak Closures . . . . .	146
5.7	The Uniqueness Case . . . . .	147
6.	Groups of $GF(2)$ -type and Related Problems by <i>Franz</i> <i>Timmesfeld</i>	
6.1	Introduction . . . . .	151
6.2	Notation and Preliminary Results . . . . .	152
6.3	Statement of Results . . . . .	156
6.4	Some Lemmas on $TI$ -Subgroups . . . . .	160
6.5	Some General Lemmas under Hypothesis A. . . . .	163
6.6	$Q$ of Symplectic Type but not Extraspecial . . . . .	167
6.7	The Case in which $z$ Is Weakly Closed in $Q$ . . . . .	169
6.8	The Proof of Theorem 3.5 . . . . .	171
6.9	The Identification of Groups with $F^*(C(z))$ Extraspecial . . . . .	174
6.10	The Theorems on $TI$ -Subgroups . . . . .	177
7.	Quasithin Groups by <i>Geoffrey Mason</i>	
7.1	Introduction . . . . .	181
7.2	The Groups . . . . .	181
7.3	The Main Result . . . . .	182
7.4	Subgroups of Parabolic Type . . . . .	183
7.5	First Main Reduction . . . . .	186
7.6	A Characterization of $J_4$ . . . . .	189
7.7	Rank Two Elements of $\mathcal{P}_f(T)$ . . . . .	189
7.8	Construction of Some Groups of Lie Type of Rank Two . . . . .	190
7.9	Level Two Analysis of $Chev(2)$ . . . . .	194
7.10	Remaining Level Two Analysis . . . . .	195
7.11	Concluding Remarks . . . . .	196
7.12	Appendix . . . . .	197

8. Odd Standard Form Problems by <i>Robert L. Griess, Jr.</i>	
8.1 Introduction . . . . .	199
8.2 Statement of Results When $m_{2,3}(G) \leq 3$ . . . . .	200
8.3 Statement of Results When $e(G) \geq 4$ . . . . .	202
8.4 Appendix . . . . .	204
9. The Revision Project and Pushing-Up by <i>George Glauberman</i>	
9.1 The Revision Project and the Odd Order Paper . . . . .	207
9.2 Pushing-Up. . . . .	213
9.3 A Pair of Characteristic Subgroups for Pushing-Up . . . . .	217
9.4 An Application of Pushing-Up to Transfer . . . . .	220
10. Pushing-Up in Finite Groups by <i>David M. Goldschmidt</i>	
10.1 Concepts and Results. . . . .	225
10.2 Local Geometries . . . . .	228
References for Part A . . . . .	231

## Part B: Representation Theory of Groups of Lie Type

11. Complex Representation Theory of Finite Groups of Lie Type by <i>R. W. Carter</i>	
11.1 The Groups . . . . .	243
11.2 Lang's Map . . . . .	244
11.3 $\ell$ -Adic Cohomology Modules . . . . .	245
11.4 Orthogonality Relations . . . . .	246
11.5 Character Formulae . . . . .	247
11.6 Geometric Conjugacy Classes . . . . .	248
11.7 Duality . . . . .	249
11.8 Semisimple Characters and Regular Characters . . . . .	251
11.9 The Brauer Complex . . . . .	252
11.10 Unipotent Characters . . . . .	254
11.11 Cuspidal Unipotent Characters . . . . .	255
References . . . . .	257
12. Modular Representations of Finite Groups of Lie Type by <i>J. E. Humphreys</i>	
Introduction . . . . .	259
12.1 Irreducible Modules . . . . .	259
12.2 Projective Modules . . . . .	270
12.3 Relation with Ordinary Characters . . . . .	278
Appendix . . . . .	286
References . . . . .	288

13. Weyl Modules for Groups of Lie Type by <i>Jens C. Jantzen</i>	
13.1 Introduction . . . . .	291
13.2 Contravariant Forms. . . . .	291
13.3 Tensor Products . . . . .	295
References . . . . .	300
<b>Part C: Geometry and the Sporadic Groups</b>	
14. Four Presentations of Leech's Lattice by <i>J. Tits</i>	
14.1 Introduction . . . . .	303
14.2 Formulae . . . . .	304
14.3 On Conway's Uniqueness Proof. . . . .	305
Acknowledgement . . . . .	307
References . . . . .	307
15. Buildings and Buekenhout Geometries by <i>J. Tits</i>	
15.1 Complex Analytic Groups . . . . .	309
15.2 Chevalley Groups and Algebraic Groups . . . . .	311
15.3 Geometries of Type $M$ and Buildings. . . . .	312
15.4 Buekenhout Geometries . . . . .	315
15.5 Sporadic Groups . . . . .	316
15.6 Miscellaneous Remarks . . . . .	318
References . . . . .	319
16. Fixed Point Free Involutions and Finite Projective Planes by <i>John G. Thompson</i>	
16.1 Ovals. . . . .	321
16.2 Permutation Modules and Cohomology . . . . .	322
16.3 Unimodular Lattices . . . . .	325
16.4 The Character of $M$ . . . . .	326
16.5 The Character of $S_n$ on $K$ . . . . .	328
16.6 $H^1(K)$ is Cyclic of Order $c(\lambda)$ . . . . .	330
16.7 Two Special Cases . . . . .	335
17. How to Construct a Baby Monster by <i>Charles C. Sims</i> . . . . .	339
References . . . . .	345