

Contents

	Page
Introduction	vii
Note on the Projects	viii
Chapter 1 Permutation Groups	1
1.1 Preliminary definitions	1
1.2 Counting principles	3
1.3 Transitivity	5
1.4 Applications to group theory	9
1.5 Extensions of multiply transitive groups	11
1.6 Primitivity	13
1.7 Regular normal subgroups	17
1.8 Project: Proof of Sylow's theorem	20
1.9 Project: Some multiply transitive groups	21
Notes and references	23
Chapter 2 Finite Geometries	24
2.1 Introduction	24
2.2 Finite fields	26
2.3 Finite vector spaces	28
2.4 The structure of $GL(V)$ and $SL(V)$	30
2.5 Projective spaces and their groups	37
2.6 More about projective spaces	41
2.7 The classical simple groups	45
2.8 Project: Near-fields and sharply 2-transitive groups	49
2.9 Project: Uniqueness of $PG(2, 4)$	51
2.10 Project: A unitary polarity in $PG(2, 9)$	51
Notes and references	52

Chapter 3	Designs	54
3.1	Four fundamental problems	54
3.2	Designs	55
3.3	Symmetric designs	59
3.4	Automorphisms of designs	65
3.5	Extensions of designs	66
3.6	Mathieu groups and associated designs	70
3.7	Project: Hadamard matrices and designs	74
3.8	Project: A $5 - (28, 7, 1)$ design	76
3.9	Project: Uniqueness of the $3 - (22, 6, 1)$ design	76
	Notes and references	78
Chapter 4	Groups and Graphs	80
4.1	Permutation groups and graphs	80
4.2	Automorphisms of graphs	85
4.3	Rank 3 groups and the associated graphs	87
4.4	Feasibility conditions for strongly regular graphs	89
4.5	The Higman-Sims group	93
4.6	Project: Some graphs and their automorphism groups	99
4.7	Project: Strongly regular graphs and biplanes	100
	Notes and references	101
Chapter 5	Maps	103
5.1	Maps and surfaces	103
5.2	Automorphisms of maps	109
5.3	Cayley graphs and Cayley maps	115
5.4	Complete maps and a theorem of Frobenius	122
5.5	Symmetrical maps	130
5.6	Project: Generalized Cayley maps	132
5.7	Project: Paley maps	134
5.8	Project: Symmetrical Cayley maps	135
	Notes and references	136
Index		138