

Contents

Introduction	1
1. Local Analysis and the Four Phases of the Classification	11
1.1. From Character Theory to Local Analysis	11
1.2. Internal Geometric Analysis	22
1.3. Why the Extreme Length?	25
1.4. Some Standard Terminology and Results	28
1.5. The Shape of the Proof	43
1.6. The Four Phases of the Classification	54
1.7. Consequences of the Classification	55
1.8. The Future of Finite Group Theory	58
2. The Known Simple Groups	61
2.1. The Groups of Lie Type	61
2.2. The Mathieu Groups	78
2.3. Janko's First Group	81
2.4. Sporadic Groups from Centralizers of Involutions	85
2.5. Computer Construction of Sporadic Groups	96
2.6. Sporadic Groups and Rank 3 Permutation Groups	104
2.7. Janko's Group J_4	110
2.8. Transpositions and the Fischer Groups	112
2.9. The Leech Lattice and the Conway Groups	120
2.10. The Fischer–Griess Group F_1	126
2.11. The List of Known Simple Groups and Their Orders	134
2.12. Statement of the General Classification Theorem	136
3. Recognition Theorems	137
3.1. The Groups of Lie Type	138
3.2. Doubly Transitive Groups	146

3.3.	The Alternating Groups	169
3.4.	The Sporadic Groups	170
4.	General Techniques of Local Analysis	177
4.1.	Solvable Groups	177
4.2.	Strong Embedding	186
4.3.	Signalizer Functors	192
4.4.	k -Balanced Groups	202
4.5.	L -Balance	210
4.6.	p -Fusion	215
4.7.	Stability and Characteristic Subgroups For Odd Primes	225
4.8.	The Bender Method, Small Class Sylow 2-Subgroups, Strong Closure, and the $p^a q^b$ -Theorem	234
4.9.	Product Fusion and Strong Closure	244
4.10.	Weak Closure and Trivial Intersection Sets	250
4.11.	Factorizations and $3'$ -Groups	253
4.12.	Failure of Thompson Factorization	260
4.13.	Pushing-up, Aschbacher Blocks, and the Local $C(G; T)$ -Theorem	266
4.14.	Properties of K -Groups: Generalities	282
4.15.	Properties of K -Groups: Specifics	295
	BIBLIOGRAPHY	315
	INDEX	327