

CONTENTS

Chapter 5. Algebraic K-theory	1
§38. Grothendieck groups	2
§38A. <i>Grothendieck groups. Frobenius functors</i>	2
§38B. <i>Grothendieck groups and projective class groups</i>	14
§38C. <i>Regular rings</i>	19
§38D. <i>Localization sequences</i>	31
§39. Grothendieck groups of integral group rings	44
§39A. <i>Localization sequences</i>	45
§39B. <i>Explicit calculations</i>	54
§40. Whitehead groups	61
§40A. <i>Introduction</i>	61
§40B. <i>Localization sequences</i>	65
§40C. <i>Elementary matrices</i>	73
§40D. <i>Unimodular rows and stably free modules</i>	77
§41. Basic elements, stable range, and cancellation	83
§42. Mayer-Vietoris sequences	101
§43. K -theory of polynomial rings	112
§44. Relative K -theory	120
§45. SK_1 of orders	138
§45A. <i>Reduced norms</i>	138
§45B. <i>Maximal orders</i>	142
§45C. <i>Finiteness of SK_1</i>	151
§45D. <i>Profinite groups</i>	156
§46. Whitehead groups of integral group rings	163
§47. Milnor's K_2 -group	184
§47A. <i>Steinberg groups and K_2</i>	184
§47B. <i>Relative K-theory</i>	190
§47C. <i>Symbols</i>	197
§48. SK_1 of integral group rings	210

Chapter 6. Class groups of integral group rings and orders	216
§ 49. Locally free class groups	217
§ 49A. <i>Basic formulas</i>	217
§ 49B. <i>Functorial properties and the kernel group</i>	229
§ 49C. <i>Frobenius functor properties for class groups of group rings</i>	238
§ 50. Class groups of integral group rings	243
§ 50A. <i>Cyclic groups of squarefree order</i>	243
§ 50B. <i>The kernel group for p-groups</i>	254
§ 50C. <i>Metacyclic groups</i>	259
§ 50D. <i>Dihedral and quaternion 2-groups</i>	266
§ 50E. <i>An involution on class groups and kernel groups</i>	274
§ 50F. <i>Cyclic p-groups</i>	283
§ 50G. <i>Twisted group rings and crossed-product orders</i>	291
§ 51. Jacobinski's Cancellation Theorem and the Eichler condition	303
§ 51A. <i>The Eichler condition</i>	304
§ 51B. <i>The Eichler-Swan Theorem</i>	306
§ 51C. <i>Locally free cancellation</i>	322
§ 52. The Hom description of the class group	329
§ 53. The Swan subgroup of the class group	343
§ 53A. <i>The Swan subgroup</i>	343
§ 53B. <i>Rings of integers in tame extensions</i>	351
§ 53C. <i>Generalized Swan subgroups</i>	353
§ 54. p -Adic logarithms and Taylor's Theorem	356
§ 55. Picard groups	369
§ 55A. <i>Basic properties</i>	369
§ 55B. <i>Picard groups of orders</i>	376
§ 55C. <i>Locally free Picard groups</i>	382
§ 55D. <i>Radical reduction</i>	391
§ 55E. <i>Picard groups of group rings</i>	396
Chapter 7. The theory of blocks	406
§ 56. Introduction to block theory	407
§ 56A. <i>Background and notation for block theory</i>	407
§ 56B. <i>Definition of p-blocks for a finite group G</i>	412
§ 56C. <i>A criterion for P.I.M.'s to belong to the same p-block</i>	414
§ 56D. <i>Central characters and blocks of KG-modules</i>	416
§ 56E. <i>The defect of a block</i>	422

§ 57. The defect group of a p -block	429
§ 57A. G -algebras, the trace map, and defect groups	429
§ 57B. Defect groups as vertices	437
§ 57C. Defect groups as Sylow intersections	440
§ 58. The Brauer Correspondence	445
§ 58A. The Brauer map	445
§ 58B. Brauer's First Main Theorem	448
§ 58C. The Brauer Correspondence	451
§ 59. Applications of blocks to character theory	462
§ 59A. The Nagao Decomposition	463
§ 59B. Brauer's Second Main Theorem	467
§ 60. p -Sections and characters in blocks	471
§ 60A. Block orthogonality and p -sections	471
§ 60B. Determination of the principal block using block orthogonality	473
§ 60C. Applications to the classification of transitive permutation groups of degree p	478
§ 61. Refinements of the Brauer Correspondence	484
§ 61A. Blocks and normal subgroups	484
§ 61B. An extension of Brauer's First Main Theorem	489
§ 61C. Brauer's Third Main Theorem	494
§ 62. Blocks with cyclic defect groups	495
§ 62A. Preliminary results from homological algebra	496
§ 62B. Functorial properties of the Green Correspondence	499
§ 62C. Uniserial algebras and blocks of finite representation type	504
§ 62D. Modular representations in blocks with cyclic defect groups	512
§ 62E. Periodic projective resolutions in blocks with cyclic defect groups	522
§ 63. Applications to group theory	530
§ 63A. The kernel of the principal block	530
§ 63B. The Brauer-Suzuki Theorem on quaternion Sylow 2-subgroups	532
§ 63C. Glauberman's Z^* -Theorem	545
Chapter 8. The representation theory of finite groups of Lie type	549
§ 64. Root systems and finite reflection groups	550
§ 64A. Finite groups generated by reflections. Root systems	550
§ 64B. Coxeter groups	561
§ 64C. Parabolic subgroups of finite Coxeter groups	570

§ 65. Finite groups with BN -pairs	576
§ 65A. <i>The Bruhat decomposition</i>	576
§ 65B. <i>Examples of BN-pairs</i>	580
§ 65C. <i>Parabolic subgroups of finite groups with BN-pairs</i>	583
§ 66. Homology representations of finite groups with BN -pairs	586
§ 66A. <i>Homology representations of finite groups</i>	586
§ 66B. <i>The Coxeter poset of a finite g.g.r.</i>	600
§ 66C. <i>The combinatorial building and the Steinberg representation of a finite group with a BN-pair</i>	605
§ 67. The Hecke algebra $\mathcal{H}(G, B)$ and the decomposition of $(1_B)^G$	609
§ 67A. <i>The structure of the Hecke algebra $\mathcal{H}(G, B)$</i>	609
§ 67B. <i>The sign representation of \mathcal{H} and the Steinberg representation of G</i>	614
§ 67C. <i>Representations of the Hecke algebra \mathcal{H} for a BN-pair of rank 2</i>	619
§ 67D. <i>The Feit-Higman Theorem on generalized polygons</i>	623
§ 67E. <i>The reflection representation of the Hecke algebra \mathcal{H}</i>	630
§ 68. Generic algebras and finite Coxeter groups	635
§ 68A. <i>Generic algebras and the Deformation Theorem</i>	635
§ 68B. <i>Parametrization of characters in $(1_B)^G$</i>	643
§ 68C. <i>Generic degrees</i>	648
§ 69. Finite groups with split BN -pairs	653
§ 69A. <i>The Levi Decomposition</i>	653
§ 69B. <i>Intersections of parabolic subgroups</i>	662
§ 70. Cuspidal characters	666
§ 70A. <i>Generalized restriction and induction</i>	666
§ 70B. <i>The philosophy of cusp forms</i>	676
§ 70C. <i>Formulas for character values</i>	681
§ 71. A Duality Operation in $\text{ch } CG$.	688
§ 71A. <i>Definition and basic properties of D_G</i>	689
§ 71B. <i>The effects of D_G on character degrees</i>	692
§ 71C. <i>The values of the Steinberg character</i>	697
§ 72. Modular representations of finite groups of Lie type	700
§ 72A. <i>The Ballard-Lusztig Theorem on characters of P.I.M.'s</i>	700
§ 72B. <i>The simple kG-modules</i>	706
Chapter 9. Rationality questions	719
§ 73. Unitary, orthogonal, and symplectic CG -modules	720
§ 73A. <i>Rationality questions over the real field \mathbb{R}</i>	720

§ 73B. <i>Induction theorems for real-valued characters</i>	727
§ 74. The Schur Index	732
§ 74A. <i>General theory</i>	732
§ 74B. <i>Schur indices for group algebras</i>	740
§ 74C. <i>The Benard-Schacher Theorem</i>	746
§ 75. Representations and characters of the symmetric group	762
§ 75A. <i>Specht modules and simple FS_n-modules</i>	762
§ 75B. <i>Solomon's Theorem and the irreducible characters of S_n</i>	774
§ 76. The Artin exponent	782
Chapter 10. Indecomposable modules	790
§ 77. Representations of graphs and Gabriel's Theorem	790
§ 77A. <i>Representations of graphs and Coxeter functors</i>	790
§ 77B. <i>Representation categories of finite type (Gabriel's Theorem)</i>	799
§ 78. Auslander-Reiten sequences	806
§ 78A. <i>The Heller loop-space operator</i>	807
§ 78B. <i>Auslander-Reiten sequences for group algebras</i>	815
§ 78C. <i>Auslander-Reiten sequences for algebras</i>	822
§ 79. Algebras of finite representation type	830
Chapter 11. The Burnside ring and the representation ring of a finite group	837
§ 80. Permutation representations and Burnside rings	838
§ 80A. <i>Burnside rings</i>	838
§ 80B. <i>G-sets and induction maps</i>	846
§ 80C. <i>Tensor induction and algebraic maps</i>	852
§ 80D. <i>Conlon's Induction Theorem</i>	859
§ 81. Representation rings	868
§ 81A. <i>Preliminary results</i>	869
§ 81B. <i>Conlon's Theorems</i>	878
§ 81C. <i>Species</i>	891
§ 81D. <i>Dual elements in the Green algebra</i>	898
§ 81E. <i>Semisimplicity of representation algebras</i>	906
§ 81F. <i>Nilpotent elements in representation algebras</i>	912
Bibliography	921
Notation index	943
Subject index	947