CONTENTS

PREFACE vii

CHAPTER I 1

1. Preliminaries 1
2. Module constructions 4
3. Finiteness conditions 5
4. Projective and relatively projective modules 8
5. Complete reducibility 15
6. The radical 17
7. Idempotents and blocks 21
8. Rings of endomorphisms 23
9. Completeness 28
10. Local rings 33
11. Unique decompositions 36
12. Criteria for lifting idempotents 38
13. Principal indecomposable modules 42
14. Duality in algebras 46
15. Relatively injective modules for algebras 50
16. Algebras over fields 53
17. Algebras over complete local domains 63
18. Extensions of domains 69
19. Representations and traces 74

CHAPTER II 79

1. Group algebras 79
2. Modules over group algebras 79
3. Relative traces 87
4. The representation algebra of $R[G]$ 92
5. Algebraic modules 93
6. Projective resolutions 95

CHAPTER III 97

1. Basic assumptions and notation 97
2. $F[G]$ modules 98
3. Group rings over complete local domains 104
4. Vertices and sources 111
5. The Green correspondence 115
6. Defect groups 123
7. Brauer homomorphisms 128
8. $R[G \times G]$ modules 131
9. The Brauer correspondence 136

CHAPTER IV 140

1. Characters 140
2. Brauer characters 142
3. Orthogonality relations 144
4. Characters in blocks 149
5. Some open problems 165
6. Higher decomposition numbers 171
7. Central idempotents and characters 178
8. Some natural mappings 181
9. Schur indices over $\mathbf{Q}_p$ 185
10. The ring $A_2^0(\bar{R}[G])$ 186
11. Self dual modules in characteristic 2 188

CHAPTER V 192

1. Some elementary results 192
2. Inertia groups 195
3. Blocks and normal subgroups 198
4. Blocks and quotient groups 202
5. Properties of the Brauer correspondence 206
6. Blocks and their germs 209
7. Isometries 215
8. π-heights 226
9. Subsections 230
10. Lower defect groups 240
11. Groups with a given deficiency class 245

CHAPTER VI 248

1. Blocks and extensions of R 248
2. Radicals and normal subgroups 249
3. Serial modules and normal subgroups 253
4. The radical of $\bar{R}[G]$ 256
5. The radical of $R[G]$ 262
6. p-radical groups 265

CHAPTER VII 269

1. Blocks with a cyclic defect group 269
2. Statements of results 273
3. Some preliminary results 281
4. Proofs of (2.1)–(2.10) 289
5. Proofs of (2.11)–(2.17) in case $K = \hat{K}$ 290
6. The Brauer tree 299
7. Proofs of (2.11)–(2.19) 301
8. Proofs of (2.20)–(2.25) 302
9. Some properties of the Brauer tree 305
10. Some consequences 308
11. Some examples 317
12. The indecomposable $\bar{R}[G]$ modules in B 322
13. Schur indices of irreducible characters in $\hat{B}$ 333
14. The Brauer tree and field extensions 336
15. Irreducible modules with a cyclic vertex 337

CHAPTER VIII 340

1. Groups with a Sylow group of prime order 340
2. Tensor products of $\bar{R}[N]$ modules 341
3. Groups of type $L_2(p)$ 347
4. A characterization of some groups 351
5. Some consequences of (4.1) 358
6. Permutations groups of prime degree 361
7. Characters of degree less than $p-1$ 365
8. Proof of (7.1) 371
9. Proof of (7.2) 380
10. Proof of (7.3) 381
11. Some properties of permutation groups 384
12. Permutation groups of degree $2p$ 387
13. Characters of degree p 391

CHAPTER IX 396

1. The structure of $A(G)$ 397
2. $A(G)$ in case a S_p-group of G is cyclic and R is a field 402
3. Permutation modules 405
4. Endo-permutation modules for p-groups 407

CHAPTER X 411

1. Groups with a normal p'-subgroup 411
2. Brauer characters of p-solvable groups 419
3. Principal indecomposable characters of p-solvable groups 421
4. Blocks of p-solvable groups 424
5. Principal series modules for p-solvable groups 427

6. The problems of Chapter IV, section 5 for p-solvable groups 428
7. Irreducible modules for p-solvable groups 433
8. Isomorphic blocks 440

CHAPTER XI 444

1. An analogue of Jordan's theorem 444

CHAPTER XII 450

1. Types of blocks 450
2. Some properties of the principal block 455
3. Involutions and blocks 456
4. Some computations with columns 459
5. Groups with an abelian S_2-group of type $(2^m, 2^m)$ 463
6. Blocks with special defect groups 465
7. Groups with a quaternion S_2-group 467
8. The $\mathbf{Z}^*$-theorem 471

BIBLIOGRAPHY 476

SUBJECT INDEX 500