

Contents

PREFACE	iii
Chapter 1. ELEMENTS OF HOMOLOGICAL ALGEBRA	1
1. Modules	1
2. Tensor Products	3
3. Existence of Tensor Products	4
4. The Homomorphism $f \otimes g$	7
5. Tensor Product of Algebras	9
6. Trilinear Maps	11
7. Exactness	12
8. Homology and Cohomology	18
9. Homotopy	23
10. Direct and Inverse Systems	25
Chapter 2. COHOMOLOGY OF FINITE GROUPS	32
11. Group Rings	32
12. The Trace Map	34

13. G -Induced Modules	37
14. Complete Resolutions	38
15. The Dependence of $H^r(G, M)$ on the G -Module M	41
16. Existence of Complete Resolutions	46
17. The Uniqueness of Cohomology Groups	51

Chapter 3. COMPUTATIONS 55

18. The Computation of $H^{-1}(G, M)$	55
19. The Standard Complete Resolution	57
20. Extensions	61
21. The Negative Cochain Complex	68
22. Homology and Cohomology of Monoids	74
23. Cohomology of Cyclic Groups	79
24. Trivial Action	84

Chapter 4. LOWER CENTRAL SERIES AND DIMENSION SUBGROUPS 87

25. Dimension Subgroups	87
26. The Additive Group of Rationals mod 1	94
27. Polynomial Maps	96
28. Polynomial 2-Cocycles	99
29. Proof of $D_2(G) = G_2$	102
30. Remarks	105

Chapter 5. RELATIONS WITH SUBGROUPS AND QUOTIENT GROUPS 107

31. Induced Homomorphisms	107
32. The Restriction and Corestriction (or Transfer) Maps	115
33. Sylow Subgroups	119
34. Generalization of G -Induced Modules	120
35. The Inflation Map	122
36. Cohomological Triviality	127
37. Cohomological Equivalence	130

Chapter 6. FINITE p -NILPOTENT GROUPS 135

38. Outer Automorphisms of Finite p -Groups	135
39. Cohomological Characterization of Finite p -Nilpotent Groups	139
40. Stable Elements	141
41. Frobenius Groups	143

Chapter 7. CUP PRODUCTS	147
42. Definition of Cup Product	147
43. Properties of Cup Products	151
44. Uniqueness	152
45. Further Properties of Cup Products	155
46. Pairings	156
47. Duality	158
48. Periodicity	163
Chapter 8. SPECTRAL SEQUENCES	167
49. Differential Groups	167
50. How Spectral Sequences Are Used	175
51. The Cartan-Leray Spectral Sequence	178
52. The Spectral Sequence of a Group Extension	181
53. The Hochschild-Serre Spectral Sequence	183
54. Exact Sequences Involving the Terms of Spectral Sequences	189
55. A Decomposition of $H^p(G, A)$ by Means of a Normal Hall Subgroup of G	190
56. The Five-Term Exact Sequence of Hochschild and Serre	192
57. Homology Spectral Sequences	194
Chapter 9. DESCENDING CENTRAL SERIES	196
58. A Homological Condition for Isomorphism of Nilpotent Groups	196
59. Existence of a Normal p -Complement	199
60. A Condition for the Existence of Free Subgroups	201
61. Presentations and Free Subgroups	203
62. The Schur Multiplier	206
Chapter 10. GALOIS GROUPS	214
63. Galois Groups	214
64. Abelian Galois Groups	222
65. Class Formations	227
BIBLIOGRAPHY	231
INDEXES	237