

CONTENTS

Preface	vii
Foreword	ix
I Introduction	1
1 Sobolev inequalities in \mathbb{R}^n	1
2 Sobolev inequalities and the heat equation on Lie groups	3
3 Harnack's principle	5
4 A guide to this book	6
II Dimensional inequalities for semigroups of operators on the L^p spaces	8
1 Introduction, notation	8
2 Hardy–Littlewood–Sobolev theory	9
3 Converses to the Hardy–Littlewood–Sobolev theory	12
4 Localizations	17
5 Symmetric submarkovian semigroups	20
References and Comments	25
III Systems of vector fields satisfying Hörmander's condition	26
1 Hörmander's condition and hypoellipticity	26
Uniformity matters	27
2 Harnack inequalities	28
Uniformity matters	32
3 The exponential map	34
4 Carnot–Carathéodory distances	39
References and Comments	41
IV The heat kernel on nilpotent Lie groups	42
1 Some remarkable properties of nilpotent Lie groups	42
2 Examples	43
3 Harnack inequalities for nilpotent Lie groups	45
4 Estimates of the heat kernel	48
5 Estimates of the volume	50
6 Sobolev's theorem	55
7 Sobolev inequalities	56
References and Comments	61
V Local theory for sums of squares of vector fields	63
1 Estimates of the volume	63
2 Proof of the Key Lemma	65
3 Local scaling of the Harnack inequality	67

4	The case of unimodular Lie groups	69
5	The general case	70
	References and Comments	72
VI	Convolution powers on finitely generated groups	74
1	Introduction	74
2	Distance and volume growth function on a group	75
3	The main results for superpolynomial groups	77
4	Comparison of Dirichlet forms and finite variance	81
5	Nilpotent finitely generated groups	84
6	Kesten's conjecture	85
	References and Comments	86
VII	Convolution powers on unimodular compactly generated groups	88
1	Main results	88
2	Dimension theory for symmetric submarkovian operators	89
3	Comparison of Dirichlet forms	94
4	Volume growth and polynomial decay of convolution powers	99
5	The case of superpolynomial growth	103
	References and Comments	105
VIII	The heat kernel on unimodular Lie groups	106
1	Preliminaries	106
2	Polynomial growth Lie groups	107
3	Harnack inequalities for polynomial growth groups	114
4	Exponential growth Lie groups	119
	References and Comments	122
IX	Sobolev inequalities on non-unimodular Lie groups	123
1	Local theory	123
2	An inequality of Hardy and some consequences	128
3	A Sobolev inequality again	132
	References and Comments	137
X	Geometric applications	138
1	Geometry of Lie groups and quasiregular maps	138
2	Picard theorems on Lie groups	139
3	Brownian motion on covering manifolds and random walks on groups	141
4	Dimension at infinity of a covering manifold	143
5	Quasiregular maps and compact manifolds	146
	References and Comments	147
	Bibliography	148
	Index	156