

Contents

<i>Preface</i>	xiii
1 Lie groups, fibre bundles and Cartan calculus	1
1.1 Introduction: Lie groups and actions of a Lie group on a manifold	2
1.2 Left- (X^L) and right- (X^R) invariant vector fields on a Lie group G	8
1.3 A summary of fibre bundles	12
1.4 Differential forms and Cartan calculus: a review	31
1.5 De Rham cohomology and Hodge-de Rham theory	43
1.6 The dual aspect of Lie groups: invariant differential forms. Invariant integration measure on G	54
1.7 The Maurer-Cartan equations and the canonical form on a Lie group G . Bi-invariant measure	58
1.8 Left-invariance and bi-invariance. Bi-invariant metric tensor field on the group manifold	62
1.9 Applications and examples for Lie groups	65
1.10 The case of super Lie groups: the supertranslation group as an example	70
1.11 Appendix A: some homotopy groups	73
1.12 Appendix B: the Poincaré polynomials of the compact simple groups	78
<i>Bibliographical notes for chapter 1</i>	81
2 Connections and characteristic classes	84
2.1 Connections on a principal bundle: an outline	84
2.2 Examples of connections	95
2.3 Equivariant forms on a Lie group	101
2.4 Characteristic classes	104
2.5 Chern classes and Chern characters	111
2.6 Chern-Simons forms of the Chern characters	115
2.7 The magnetic monopole	120

2.8	Yang-Mills instantons	126
2.9	Pontryagin classes and the Euler class	131
2.10	Index theorems for manifolds without boundary	137
2.11	Index theorem for the spin complex. Twisted complexes	144
	<i>Bibliographical notes for chapter 2</i>	149
3	A first look at cohomology of groups and related topics	152
3.1	Some known facts of 'non-relativistic' mechanics: two-cocycles	152
3.2	Projective representations of a Lie group: a review of Bargmann's theory	159
3.3	The Weyl-Heisenberg group and quantization	163
3.4	The extended Galilei group	168
3.5	The two-cocycle ambiguity and the Bargmann cocycle for the Galilei group	170
3.6	Dynamical groups and symplectic cohomology: an introduction	173
3.7	The adjoint and the coadjoint representations of the extended Galilei group and its algebra	181
3.8	On the possible failure of the associative property: three-cocycles	186
3.9	Some remarks on central extensions	189
3.10	Contractions and group cohomology	192
	<i>Bibliographical notes for chapter 3</i>	197
4	An introduction to abstract group extension theory	199
4.1	Exact sequences of group homomorphisms	199
4.2	Group extensions: statement of the problem in the general case	201
4.3	Principal bundle description of an extension $\tilde{G}(K, G)$	207
4.4	Characterization of an extension through the factor system ω	209
4.5	Group law for \tilde{G} in terms of the factor system	211
	<i>Bibliographical notes for chapter 4</i>	213
5	Cohomology groups of a group G and extensions by an abelian kernel	215
5.1	Cohomology of groups	215
5.2	Extensions \tilde{G} of G by an abelian group A	221
5.3	An example from supergroup theory: superspace as a group extension	223
5.4	$\mathcal{F}(M)$ -valued cochains: cohomology induced by the action of G on a manifold M^m	225
	<i>Bibliographical notes for chapter 5</i>	229
6	Cohomology of Lie algebras	230
6.1	Cohomology of Lie algebras: general definitions	230
6.2	Extensions of a Lie algebra \mathcal{G} by an abelian algebra $\mathcal{A} : H_p^2(\mathcal{G}, \mathcal{A})$	234

6.3	Three cases of special interest	236
6.4	Local exponents and the isomorphism between $H_0^2(G, R)$ and $H_0^2(\mathcal{G}, R)$	238
6.5	Cohomology groups for semisimple Lie algebras. The Whitehead lemma	242
6.6	Higher cohomology groups	244
6.7	The Chevalley-Eilenberg formulation of the Lie algebra cohomology and invariant differential forms on a Lie group G	246
6.8	The BRST approach to the Lie algebra cohomology	249
6.9	Lie algebra cohomology vs. Lie group cohomology	253
6.10	$\mathcal{F}(M)$ -valued Lie algebra cohomology	262
	<i>Bibliographical notes for chapter 6</i>	262
7	Group extensions by non-abelian kernels	264
7.1	The information contained in the G -kernel (K, σ)	264
7.2	The necessary and sufficient condition for a G -kernel (K, σ) to be extendible	268
7.3	Construction of extensions	270
7.4	Example: the covering groups of the complete Lorentz group L	275
7.5	A brief comment on the meaning of the higher cohomology groups $H^n(G, A)$	279
	<i>Bibliographical notes for chapter 7</i>	280
8	Cohomology and Wess-Zumino terms: an introduction	281
8.1	A short review of the variational principle and of the Noether theorem in Newtonian mechanics	281
8.2	Invariant forms on a manifold and cohomology; WZ terms	286
8.3	Newtonian mechanics and Wess-Zumino terms: two simple examples	290
8.4	Cohomology and classical mechanics: preliminaries	297
8.5	The cohomological descent approach to classical anomalies	303
8.6	A simple application: the free Newtonian particle	308
8.7	The massive superparticle and Wess-Zumino terms for supersymmetric extended objects	311
8.8	Supersymmetric extended objects and the supersymmetry algebra	315
8.9	A Lagrangian description of the magnetic monopole	324
	<i>Bibliographical notes for chapter 8</i>	329
9	Infinite-dimensional Lie groups and algebras	331
9.1	Introduction	331
9.2	The group of mappings $G(M)$ associated with a compact Lie group G , and its Lie algebra $\mathcal{G}(M)$	335
9.3	Current algebras as infinite-dimensional Lie algebras	338

9.4	The Kac-Moody or untwisted affine algebra	340
9.5	The Virasoro algebra	345
9.6	Chevalley-Eilenberg cohomology on $\text{Diff } S^1$ and two-dimensional gravity	351
9.7	The conformal algebra	354
	<i>Bibliographical notes for chapter 9</i>	358
10	Gauge anomalies	360
10.1	The group of gauge transformations and the orbit space of Yang-Mills potentials	360
10.2	Theory with Dirac fermions: the abelian anomaly and the index theorem	366
10.3	The action of gauge transformations on the space of functionals	372
10.4	Cohomology on $G(M)$, $\mathcal{G}(M)$ and its BRST formulation	374
10.5	Theory with Weyl fermions: non-abelian gauge anomalies and their path integral calculation	379
10.6	The non-abelian anomaly as a probe for non-trivial topology	387
10.7	The geometry of consistent non-abelian gauge anomalies	394
10.8	The cohomological descent in the trivial $P(G, M)$ case: cochains and coboundary operators	398
10.9	The descent equations. Cocycles and coboundaries	403
10.10	A compact form for the gauge algebra descent equations	407
10.11	Specific results in $D = 2, 4$ spacetime dimensions	408
10.12	On the existence of consistent anomalous theories	416
10.13	Appendix: calculating Chern-Simons forms	418
	<i>Bibliographical notes for chapter 10</i>	420
	<i>List of symbols</i>	424
	<i>References</i>	429
	<i>Index</i>	444