

# Contents

---

<b>Preface</b>	<b>vii</b>
<b>1 Elementary theory of nilpotent Lie groups and Lie algebras</b>	<b>1</b>
1.1 Basic facts about Lie groups and Lie algebras	1
1.2 Nilpotent Lie groups	10
1.3 Coadjoint orbits and the dual of $\mathfrak{g}$	25
1.4 Notes and references	36
<b>2 Kirillov theory</b>	<b>38</b>
2.1 Some generalities on representations	38
2.2 The elements of Kirillov theory	45
2.3 Some preliminary lemmas	56
2.4 Proof of the basic theorems	62
2.5 Subgroups of codimension 1 and representations	69
2.6 Notes and references	76
<b>3 Parametrization of coadjoint orbits</b>	<b>81</b>
3.1 Orbits of unipotent actions	81
3.2 Canonical subalgebras related to elements of $\mathfrak{g}^*$	98
3.3 The center of the enveloping algebra of a nilpotent Lie group	109
3.4 Notes and references	121
<b>4 Plancherel formula and related topics</b>	<b>123</b>
4.1 $C^\infty$ vectors for irreducible representations of a nilpotent Lie group	123
4.2 Traces of irreducible representations	131
4.3 The canonical measures on orbits and the Plancherel formula	144
4.4 Nilpotent Lie groups that are not simply connected	161
4.5 Square-integrable representations and flat orbits	168
4.6 Representations of $u(\mathfrak{g})$ and their infinitesimal characters	184
4.7 Notes and references	190

<b>5</b>	<b>Discrete cocompact subgroups</b>	<b>193</b>
5.1	Rational structures and uniform subgroups	193
5.2	Rational subgroups of nilpotent Lie groups	208
5.3	Fundamental domains for uniform subgroups	212
5.4	Lattice subgroups	214
5.5	Notes and References	224
<b>Appendix</b>		<b>226</b>
A.1	$C^\infty$ vectors for representations of a Lie group	226
A.2	Schwartz functions $\mathcal{S}(G)$	235
A.3	Hermite functions on $\mathbb{R}^n$	243
<b>Bibliography</b>		<b>254</b>
<b>Symbol index</b>		<b>261</b>
<b>Subject index</b>		<b>263</b>