

CONTENTS

	PREFACE	vii
CHAPTER 1	INTRODUCTION	1
	1.1 Particle on a One-Dimensional Lattice	2
	1.2 Representations of the Discrete Translation Operators	4
	1.3 Physical Consequences of Translational Symmetry	6
	1.4 The Representation Functions and Fourier Analysis	8
	1.5 Symmetry Groups of Physics	9
CHAPTER 2	BASIC GROUP THEORY	12
	2.1 Basic Definitions and Simple Examples	12
	2.2 Further Examples, Subgroups	14
	2.3 The Rearrangement Lemma and the Symmetric (Permutation) Group	16
	2.4 Classes and Invariant Subgroups	19
	2.5 Cosets and Factor (Quotient) Groups	21
	2.6 Homomorphisms	23
	2.7 Direct Products	24
	Problems	25
CHAPTER 3	GROUP REPRESENTATIONS	27
	3.1 Representations	27
	3.2 Irreducible, Inequivalent Representations	32
	3.3 Unitary Representations	35
	3.4 Schur's Lemmas	37
	3.5 Orthonormality and Completeness Relations of Irreducible Representation Matrices	39
	3.6 Orthonormality and Completeness Relations of Irreducible Characters	42
	3.7 The Regular Representation	45
	3.8 Direct Product Representations, Clebsch-Gordan Coefficients	48
	Problems	52
CHAPTER 4	GENERAL PROPERTIES OF IRREDUCIBLE VECTORS AND OPERATORS	54
	4.1 Irreducible Basis Vectors	54

	4.2 The Reduction of Vectors—Projection Operators for Irreducible Components	56
	4.3 Irreducible Operators and the Wigner-Eckart Theorem Problems	59 62
CHAPTER 5	REPRESENTATIONS OF THE SYMMETRIC GROUPS	64
	5.1 One-Dimensional Representations	65
	5.2 Partitions and Young Diagrams	65
	5.3 Symmetrizers and Anti-Symmetrizers of Young Tableaux	67
	5.4 Irreducible Representations of S_n	68
	5.5 Symmetry Classes of Tensors Problems	70 78
CHAPTER 6	ONE-DIMENSIONAL CONTINUOUS GROUPS	80
	6.1 The Rotation Group $SO(2)$	81
	6.2 The Generator of $SO(2)$	83
	6.3 Irreducible Representations of $SO(2)$	84
	6.4 Invariant Integration Measure, Orthonormality and Completeness Relations	86
	6.5 Multi-Valued Representations	88
	6.6 Continuous Translational Group in One Dimension	89
	6.7 Conjugate Basis Vectors Problems	91 93
CHAPTER 7	ROTATIONS IN THREE-DIMENSIONAL SPACE—THE GROUP $SO(3)$	94
	7.1 Description of the Group $SO(3)$	94
	7.1.1 The Angle-and-Axis Parameterization	96
	7.1.2 The Euler Angles	97
	7.2 One Parameter Subgroups, Generators, and the Lie Algebra	99
	7.3 Irreducible Representations of the $SO(3)$ Lie Algebra	102
	7.4 Properties of the Rotational Matrices $D^j(\alpha, \beta, \gamma)$	107
	7.5 Application to Particle in a Central Potential	109
	7.5.1 Characterization of States	110
	7.5.2 Asymptotic Plane Wave States	111
	7.5.3 Partial Wave Decomposition	111
	7.5.4 Summary	112
	7.6 Transformation Properties of Wave Functions and Operators	112
	7.7 Direct Product Representations and Their Reduction	117

7.8 Irreducible Tensors and the Wigner-Eckart Theorem	122
Problems	123

CHAPTER 8 THE GROUP SU(2) AND MORE ABOUT SO(3) 125

8.1 The Relationship between SO(3) and SU(2)	125
8.2 Invariant Integration	129
8.3 Orthonormality and Completeness Relations of D^j	133
8.4 Projection Operators and Their Physical Applications	135
8.4.1 Single Particle State with Spin	136
8.4.2 Two Particle States with Spin	138
8.4.3 Partial Wave Expansion for Two Particle Scattering with Spin	140
8.5 Differential Equations Satisfied by the D^j -Functions	141
8.6 Group Theoretical Interpretation of Spherical Harmonics	143
8.6.1 Transformation under Rotation	144
8.6.2 Addition Theorem	145
8.6.3 Decomposition of Products of Y_{lm} With the Same Arguments	145
8.6.4 Recursion Formulas	145
8.6.5 Symmetry in m	146
8.6.6 Orthonormality and Completeness	146
8.6.7 Summary Remarks	146
8.7 Multipole Radiation of the Electromagnetic Field	147
Problems	150

CHAPTER 9 EUCLIDEAN GROUPS IN TWO- AND THREE-DIMENSIONAL SPACE 152

9.1 The Euclidean Group in Two-Dimensional Space E_2	154
9.2 Unitary Irreducible Representations of E_2 —the Angular-Momentum Basis	156
9.3 The Induced Representation Method and the Plane-Wave Basis	160
9.4 Differential Equations, Recursion Formulas, and Addition Theorem of the Bessel Function	163
9.5 Group Contraction—SO(3) and E_2	165
9.6 The Euclidean Group in Three Dimensions: E_3	166
9.7 Unitary Irreducible Representations of E_3 by the Induced Representation Method	168
9.8 Angular Momentum Basis and the Spherical Bessel Function	170
Problems	171

CHAPTER 10	THE LORENTZ AND POINCARÉ GROUPS, AND SPACE-TIME SYMMETRIES	173
10.1	The Lorentz and Poincaré Groups	173
10.1.1	Homogeneous Lorentz Transformations	174
10.1.2	The Proper Lorentz Group	177
10.1.3	Decomposition of Lorentz Transformations	179
10.1.4	Relation of the Proper Lorentz Group to $SL(2)$	180
10.1.5	Four-Dimensional Translations and the Poincaré Group	181
10.2	Generators and the Lie Algebra	182
10.3	Irreducible Representations of the Proper Lorentz Group	187
10.3.1	Equivalence of the Lie Algebra to $SU(2) \times SU(2)$	187
10.3.2	Finite Dimensional Representations	188
10.3.3	Unitary Representations	189
10.4	Unitary Irreducible Representations of the Poincaré Group	191
10.4.1	Null Vector Case ($P_\mu = 0$)	192
10.4.2	Time-Like Vector Case ($c_1 > 0$)	192
10.4.3	The Second Casimir Operator	195
10.4.4	Light-Like Case ($c_1 = 0$)	196
10.4.5	Space-Like Case ($c_1 < 0$)	199
10.4.6	Covariant Normalization of Basis States and Integration Measure	200
10.5	Relation Between Representations of the Lorentz and Poincaré Groups—Relativistic Wave Functions, Fields, and Wave Equations	202
10.5.1	Wave Functions and Field Operators	202
10.5.2	Relativistic Wave Equations and the Plane Wave Expansion	203
10.5.3	The Lorentz-Poincaré Connection	206
10.5.4	“Deriving” Relativistic Wave Equations	208
	Problems	210
CHAPTER 11	SPACE INVERSION INVARIANCE	212
11.1	Space Inversion in Two-Dimensional Euclidean Space	212
11.1.1	The Group $O(2)$	213
11.1.2	Irreducible Representations of $O(2)$	215
11.1.3	The Extended Euclidean Group \tilde{E}_2 and its Irreducible Representations	218
11.2	Space Inversion in Three-Dimensional Euclidean Space	221

11.2.1	The Group $O(3)$ and its Irreducible Representations	221
11.2.2	The Extended Euclidean Group \tilde{E}_3 and its Irreducible Representations	223
11.3	Space Inversion in Four-Dimensional Minkowski Space	227
11.3.1	The Complete Lorentz Group and its Irreducible Representations	227
11.3.2	The Extended Poincaré Group and its Irreducible Representations	231
11.4	General Physical Consequences of Space Inversion	237
11.4.1	Eigenstates of Angular Momentum and Parity	238
11.4.2	Scattering Amplitudes and Electromagnetic Multipole Transitions	240
	Problems	243
CHAPTER 12	TIME REVERSAL INVARIANCE	245
12.1	Preliminary Discussion	245
12.2	Time Reversal Invariance in Classical Physics	246
12.3	Problems with Linear Realization of Time Reversal Transformation	247
12.4	The Anti-Unitary Time Reversal Operator	250
12.5	Irreducible Representations of the Full Poincaré Group in the Time-Like Case	251
12.6	Irreducible Representations in the Light-Like Case ($c_1 = c_2 = 0$)	254
12.7	Physical Consequences of Time Reversal Invariance	256
12.7.1	Time Reversal and Angular Momentum Eigenstates	256
12.7.2	Time-Reversal Symmetry of Transition Amplitudes	257
12.7.3	Time Reversal Invariance and Perturbation Amplitudes	259
	Problems	261
CHAPTER 13	FINITE-DIMENSIONAL REPRESENTATIONS OF THE CLASSICAL GROUPS	262
13.1	$GL(m)$: Fundamental Representations and The Associated Vector Spaces	263
13.2	Tensors in $V \times \tilde{V}$, Contraction, and $GL(m)$ Transformations	265
13.3	Irreducible Representations of $GL(m)$ on the Space of General Tensors	269

13.4	Irreducible Representations of Other Classical Linear Groups	277
13.4.1	Unitary Groups $U(m)$ and $U(m_+, m_-)$	277
13.4.2	Special Linear Groups $SL(m)$ and Special Unitary Groups $SU(m_+, m_-)$	280
13.4.3	The Real Orthogonal Group $O(m_+, m_-; \mathbb{R})$ and the Special Real Orthogonal Group $SO(m_+, m_-; \mathbb{R})$	283
13.5	Concluding Remarks	289
	Problems	290
APPENDIX I	NOTATIONS AND SYMBOLS	292
I.1	Summation Convention	292
I.2	Vectors and Vector Indices	292
I.3	Matrix Indices	293
APPENDIX II	SUMMARY OF LINEAR VECTOR SPACES	295
II.1	Linear Vector Space	295
II.2	Linear Transformations (Operators) on Vector Spaces	297
II.3	Matrix Representation of Linear Operators	299
II.4	Dual Space, Adjoint Operators	301
II.5	Inner (Scalar) Product and Inner Product Space	302
II.6	Linear Transformations (Operators) on Inner Product Spaces	304
APPENDIX III	GROUP ALGEBRA AND THE REDUCTION OF REGULAR REPRESENTATION	307
III.1	Group Algebra	307
III.2	Left Ideals, Projection Operators	308
III.3	Idempotents	309
III.4	Complete Reduction of the Regular Representation	312
APPENDIX IV	SUPPLEMENTS TO THE THEORY OF SYMMETRIC GROUPS S_n	314
APPENDIX V	CLEBSCH-GORDAN COEFFICIENTS AND SPHERICAL HARMONICS	318
APPENDIX VI	ROTATIONAL AND LORENTZ SPINORS	320
APPENDIX VII	UNITARY REPRESENTATIONS OF THE PROPER LORENTZ GROUP	328
APPENDIX VIII	ANTI-LINEAR OPERATORS	331
	REFERENCES AND BIBLIOGRAPHY	335
	INDEX	338