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- Chapter IX. Compact Groups
- Chapter X. Abelian Groups and Commutative Banach $*$ -Algebraic Bundles
- Chapter XI. Induced Representations and the Imprimitivity Theorem
- Chapter XII. The Generalized Mackey Analysis