Table of Contents

Series Editor's Preface v
Preface xix
List of Special Symbols xxi
Chapter 0: Introduction 1
Chapter 1: Elements of the Theory of Lie Groups and Lie Algebras 6

1.0. Preliminary Information from Algebra, Topology, and Functional Analysis 6
1.0.1. Groups, rings, linear spaces, Lie algebras 6
1.0.2. Subgroups and subalgebras 9
1.0.3. Homomorphisms and automorphisms 11
1.0.4. Direct products, direct sums, multilinear functionals 15
1.0.5. The matrix form of linear operators, bilinear and sesquilinear functionals 18
1.0.6. Topological spaces and manifolds 19
1.0.7. Generalized functions 22
1.0.8. Banach and Hilbert spaces 25
1.0.9. Countably Hilbert spaces and nuclear spaces 28
1.0.10. Eigenvalues and eigenvectors 30
1.0.11. Tensor product of spaces and operators 32
1.0.12. Banach algebras 34

1.1. Lie Groups and Lie Algebras 35
1.1.1. Linear groups 35
1.1.2. Lie groups 38
1.1.3. Correspondence between Lie groups and Lie algebras . . . 40
1.1.4. The exponential mapping 42
1.1.5. The adjoint Lie group. The center of the universal enveloping algebra 45
1.1.6. The Killing form 46
1.1.7. Semisimple Lie algebras and groups 48
1.1.8. Nilpotent and solvable Lie algebras and groups 49

1.2. Homogeneous Spaces with Semisimple Groups of Motions . 51
1.2.1. Homogeneous spaces 51
1.2.2. Symmetric homogeneous spaces 55
1.2.3. Cartan decomposition of semisimple Lie algebras and homogeneous spaces with semisimple motion groups 56

1.2.4. Pseudo-Riemannian symmetric homogeneous spaces 59
1.2.5. Complex homogeneous domains 61
1.2.6. Invariant measures 62
1.2.7. The group ring of a Lie group 66

Chapter 2: Group Representations and Harmonic Analysis on Groups 68

2.1. Representations of Lie Groups and Lie Algebras 68
2.1.1. Group representations 68
2.1.2. Expressions for representations in matrix form 71
2.1.3. Representations of group rings 73
2.1.4. Infinite differentiability of matrix elements of representations of Lie groups 74
2.1.5. Representations of Lie algebras and of universal enveloping algebras 75
2.1.6. Group representations by shift operators in function spaces 77

2.2. Basic Concepts of the Theory of Representations 80
2.2.1. Invariant subspaces. Cyclic, irreducible and completely reducible representations 80
2.2.2. Contragradient representations 82
2.2.3. Unitary representations 83
2.2.4. Intertwining and invariant operators 85
2.2.5. Equivalent representations 87
2.2.6. Representations of complex Lie groups and algebras and of their real forms 88
2.2.7. Characters of finite dimensional representations 89
2.2.8. Schur's lemma 90
2.2.9. Invariant bilinear and sesquilinear functionals 91
2.2.10. Intertwining operators for completely reducible representations 93

2.3. Harmonic Analysis on Groups and on Homogeneous Spaces 94
2.3.1. Introduction 94
2.3.2. The realization of irreducible representations by shift operators 95
2.3.3. One-sided invariant subspaces of $\mathfrak{M}_T$ 96
2.3.4. The Peter-Weyl theorem 97
2.3.5. The complete orthonormal system of functions on a compact group 99
2.3.6. Decomposition of the regular representation of a compact group 101

2.3.7. The approximation theorem 102
2.3.8. Spherical functions 103
2.3.9. Expansion of functions on homogeneous compact spaces 104
2.3.10. Expansion of functions on a compact group and the group ring 104
2.3.11. Expansion of central functions 107

Chapter 3: Commutative Groups and Elementary Functions. The Group of Linear Transformations of the Straight Line and the Gamma-Function. Hypergeometric Functions 109

3.1. Representations of One-Dimensional Commutative Lie Groups and Elementary Functions 109
3.1.1. Irreducible representations of the additive group of real numbers and the exponential function 109
3.1.2. Representations of the multiplicative group of positive numbers and the power function 109
3.1.3. Trigonometric functions and representations of the group T 110
3.1.4. The group of hyperbolic rotations of the plane and hyperbolic functions 111
3.1.5. The connection between the groups $SO(2)$, $SO(1,1)$ and $SO(2,\mathbf{C})$ 112
3.1.6. The generalized functions x_+^λ, x_-^λ, $|x|^\lambda$, $|x|^\lambda$ sign x, $(x \pm i0)^\lambda$ and their properties 112

3.2. The Groups $SO(2)$ and R, Fourier Series and Integrals . . 114
3.2.1. Fourier Series as expansion of functions, defined on $SO(2)$ 114
3.2.2. Expansion of infinitely differentiable functions 115
3.2.3. The Fourier transform 116
3.2.4. Decomposition of the regular representation of R 119

3.3. Fourier Transform in the Complex Domain. Mellin and Laplace Transforms 120
3.3.1. Fourier transform in the complex domain 120
3.3.2. The Laplace transform 122
3.3.3. Transformation of square-integrable functions 122
3.3.4. The Mellin transform 124
3.3.5. The Gauss-Weierstrass transform 126

3.4. Representations of the Group of Linear Transforms of the Straight Line and the Gamma-Function 127
3.4.1. The group of linear transformations of the straight line and its representations 127

3.4.2. Diagonalization of the operators $R_\lambda(g(a,0))$ 129
3.4.3. Expression for the kernel $K(w,z;g)$ in terms of the Γ-function 131
3.4.4. Properties of the Γ-function 132
3.4.5. The addition formula for the Γ-function and its corollaries 135
3.4.6. The beta-function and the multiplication formula for $\Gamma(x)$ 137
3.4.7. The Fourier transform of the functions x_+^u and x_-^u . . . 139

3.5. Hypergeometric Functions and Their Properties 141
3.5.1. Hypergeometric functions 141
3.5.2. Some properties of hypergeometric functions 142
3.5.3. Elementary properties of the hypergeometric function ${}_2F_1$ 145
3.5.4. Some integrals involving the hypergeometric function . . 147
3.5.5. Dougall's formula 148
3.5.6. Cylindrical functions 148
3.5.7. Whittaker functions, parabolic cylinder function, Laguerre and Hermite polynomials 155
3.5.8. Jacobi and Legendre polynomials and functions 156
3.5.9. Functions of discrete variable 157
3.5.10. Fractional integration 160
3.5.11. Fractional integration and special functions 161

Chapter 4: Representations of the Groups of Motions of Euclidean and Pseudo-Euclidean Planes, and Cylindrical Functions 165

4.1. Representations of the Group $ISO(2)$ and Bessel Functions with Integral Index 165
4.1.1. The Group $ISO(2)$ 165
4.1.2. Irreducible representations of $ISO(2)$ 166
4.1.3. Calculation of matrix elements of the representations T_R . 167
4.1.4. The addition theorem and the multiplication formula . . 168
4.1.5. The generating function 170

4.2. Representations of the Group $ISO(1,1)$, Macdonald and Hankel Functions 171
4.2.1. The pseudo-Euclidean plane 171
4.2.2. The group $ISO(1,1)$ 171
4.2.3. Representations of $ISO(1,1)$ 173
4.2.4. The representations T_R, Macdonald and Hankel functions 175

4.3. Functional Relations for Cylindrical Functions 179
4.3.1. Introductory remarks 179
4.3.2. Recurrence relations and differential equations 180
4.3.3. Cylindrical functions with half-integral index 181
4.3.4. Integral representations of the Barnes type 182
4.3.5. Mellin transforms 184
4.3.6. Addition theorems 188
4.3.7. Multiplication theorems 191
4.3.8. The Ramanujan formula 192
4.4. Quasi-Regular Representations of the Groups $ISO(2)$, $ISO(1,1)$ and Integral Transforms 193
4.4.1. The quasi-regular representation of the group $ISO(2)$ . . 193
4.4.2. The Fourier-Bessel transform 196
4.4.3. The quasi-regular representation of $ISO(1,1)$ 198
4.4.4. Integral transforms 200
4.4.5. Mutually reciprocal integral transform 204

Chapter 5: Representations of Groups of Third Order Triangular Matrices, the Confluent Hypergeometric Function, and Related Polynomials and Functions 207

5.1. Representations of the Group of Third Order Real Triangular Matrices 207
5.1.1. The group of third order triangular matrices 207
5.1.2. Irreducible representations of the group G 208
5.1.3. Another realization of the representations T_χ 209
5.1.4. Calculation of the kernels of the representations R_χ . . . 211
5.2. Functional Relations for Whittaker Functions 213
5.2.1. Relations between infinitesimal operators and operators of the representation R_χ 213
5.2.2. Recurrence relations 214
5.2.3. The Whittaker differential equation 217
5.2.4. The Mellin-Barnes integral representation 219
5.2.5. Symmetry relations for Whittaker functions 221
5.3. Functional Relations for the Confluent Hypergeometric Function and for Parabolic Cylinder Functions 221
5.3.1. The functions $\Phi(\alpha;\gamma;z)$ and $\Psi(\alpha;\gamma;z)$ 221
5.3.2. Recurrence relations for $\Phi(\alpha;\gamma;z)$ and $\Psi(\alpha;\gamma;z)$ 222
5.3.3. The differential equation 225
5.3.4. The connection of the functions $\Phi(\alpha;\gamma;z)$, $\Psi(\alpha;\gamma;z)$, $M_{\lambda\mu}(z)$, $W_{\lambda\mu}(z)$ with cylindrical functions 225
5.3.5. Parabolic cylinder functions 227

5.3.6. Hermite polynomials 232
5.4. **Integrals Involving Whittaker Functions and Parabolic Cylinder Functions** 236
5.4.1. The Mellin transform in parameters 236
5.4.2. Continuous addition theorems 239
5.4.3. Multiplication theorems 242
5.4.4. Degenerate cases of addition theorems 243
5.4.5. Integral transforms connected with parabolic cylinder functions and Whittaker functions 244
5.5. **Representations of the Group of Complex Third Order Triangular Matrices, Laguerre and Charlier Polynomials** . 247
5.5.1. Representations of the group of complex third order triangular matrices 247
5.5.2. Recurrence relations and the differential equation for Laguerre polynomials 252
5.5.3. The Rodrigues formula and generating functions for Laguerre polynomials 254
5.5.4. The orthogonality relation for Laguerre polynomials . . . 256
5.5.5. Summation formulas for Laguerre polynomials 257
5.5.6. Addition theorems and multiplication formulas for Laguerre polynomials 261
5.5.7. The connection btween Laguerre polynomials and cylindrical functions 264
5.5.8. Charlier polynomials 264

Chapter 6: Representations of the Groups $SU(2)$, $SU(1,1)$ and Related Special Functions: Legendre, Jacobi, Chebyshev Polynomials and Functions, Gegenbauer, Krawtchouk, Meixner Polynomials 269

6.1. **The Groups $SU(2)$ and $SU(1,1)$** 269
6.1.1. Parametrization 269
6.1.2. Lie algebras of the groups $SU(2)$ and $SU(1,1)$ 272
6.1.3. Connection with other groups 273
6.1.4. Invariant measures 274
6.1.5. The universal covering group for $SU(1,1)$ 275
6.2. **Finite Dimensional Irreducible Representations of the Groups $GL(2,\mathbb{C})$ and $SU(2)$** 276
6.2.1. Representations in spaces of homogeneous polynomials . . 276
6.2.2. Infinitesimal operators of the representation T_ℓ 277
6.2.3. The invariant scalar product 278

6.3. Matrix Elements of the Representations T_ℓ of the Group $SL(2,\mathbb{C})$ and Jacobi, Gegenbauer and Legendre Polynomials 280
6.3.1. Calculation of matrix elements 280
6.3.2. Other expressions for the matrix elements 283
6.3.3. Expressions in terms of the Euler angles 284
6.3.4. Expression for the function $P^{\ell}_{mn}(z)$ in terms of the hypergeometric series 286
6.3.5. Integral representations, the Rodrigues formula for $P^{\ell}_{mn}(z)$ 287
6.3.6. Symmetry relations and special values of $P^{\ell}_{mn}(z)$ 287
6.3.7. Connection of $P^{\ell}_{mn}(z)$ with the classical orthogonal polynomials 288
6.3.8. Some properties of Jacobi polynomials 289
6.3.9. Gegenbauer polynomials 291
6.3.10. Legendre polynomials 295
6.3.11. Legendre polynomials as zonal spherical functions 297
6.3.12. Bessel functions and Jacobi polynomials 297
6.4. Representations of the Group $SU(1,1)$ 298
6.4.1. The representations T_χ 298
6.4.2. Irreducibility at non-integral points 300
6.4.3. Representations with integral χ 301
6.4.4. Equivalent representations 303
6.4.5. Hermitian-adjoint representations 306
6.4.6. Unitary representations 307
6.4.7. Irreducible unitary representations of the group $\widetilde{SU}(1,1)$ 310
6.5. Matrix Elements of Representations of $SU(1,1)$, Jacobi and Legendre Functions 311
6.5.1. Calculation of the matrix elements of the representations T_χ 311
6.5.2. Expressions for matrix elements in terms of Euler angles . 312
6.5.3. Expression for $\mathfrak{P}^{\tau}_{mn}(\cosh t)$ in terms of the hypergeometric function 313
6.5.4. Integral representations of $\mathfrak{P}^{\tau}_{mn}(\cosh t)$ 314
6.5.5. Symmetry relations for $\mathfrak{P}^{\tau}_{mn}(\cosh t)$ 318
6.5.6. The functions $\mathfrak{P}^{\tau}_{mn}(\cosh t)$ in the integral case 320
6.5.7. Connection of $\mathfrak{P}^{\tau}_{mn}(\cosh t)$ with special functions 322
6.5.8. The Rodrigues formula for the functions $\mathfrak{P}^{\tau}_{mn}(\cosh t)$ . . 323
6.5.9. Legendre functions and associated Legendre functions . . 324
6.5.10. Zonal and associated spherical functions of the representations T_χ 325

6.6. Addition Theorems and Multiplication Formulas 326
6.6.1. Addition theorems for the functions $\mathfrak{P}^{\tau}_{mn}(\cosh t)$ 326
6.6.2. Addition theorems for Legendre functions 327
6.6.3. Multiplication formulas 327
6.6.4. Analog of the Ramanujan Formula 330
6.7. Generating Functions and Recurrence Formulas 331
6.7.1. Generating functions for fixed ℓ, τ and n 331
6.7.2. Recurrence formulas for $P^{\ell}_{mn}(z)$ and $\mathfrak{P}^{\tau}_{mn}(\cosh t)$ for fixed ℓ and τ 334
6.7.3. Recurrence relations for $P^{\ell}_{mn}(z)$ and $\mathfrak{P}^{\tau}_{mn}(\cosh t)$ with various ℓ and τ 336
6.7.4. Recurrence relations for Jacobi polynomials 338
6.7.5. The differential equations for $P^{\ell}_{mn}(z)$, $P^{(\alpha,\beta)}_{n}(z)$ and $\mathfrak{P}^{\tau}_{mn}(z)$ 341
6.7.6. The differential equation and recurrence relations for Gegenbauer polynomials 342
6.7.7. Recurrence relations for Legendre polynomials 344
6.7.8. The generating function for $P^{\ell}_{mn}(z)$ for fixed m and n . . 344
6.7.9. The continuous generating function for $\mathfrak{P}^{\tau}_{mn}(\cosh t)$ . . . 346
6.8. Matrix Elements of Representations of $SU(2)$ and $SU(1,1)$ as Functions of Column Index. Krawtchouk and Meixner Polynomials 346
6.8.1. The representations T_{ℓ} of $SU(2)$ and Krawtchouk polynomials 346
6.8.2. The discrete series representations of the group $SU(1,1)$ and Meixner polynomials 350
6.8.3. Connection of Krawtchouk and Meixner polynomials with other polynomials 353
6.8.4. Krawtchouk-Meixner functions 354
6.9. Characters of Representations of $SU(2)$ and Chebyshev Polynomials 357
6.9.1. Chebyshev polynomials 357
6.9.2. Calculation of characters of the representations T_{ℓ} of the group $SU(2)$ 358
6.9.3. Characters and Chebyshev polynomials 360
6.9.4. Expansion of functions in Chebyshev polynomials and the completeness of the system of the representations T_{ℓ} . . 361
6.9.5. The tensor product of the representations T_{ℓ} of $SU(2)$ . . 363
6.10. Expansion of Functions on the Group $SU(2)$ 364
6.10.1. Orthogonality relations for the functions $P^{\ell}_{mn}(z)$ and related polynomials 364

6.10.2. Expansions into series in $P_{mn}^{\ell}(z)$ and in related polynomials 366
6.10.3. The Laplace operator 370
6.10.4. Expansion of infinitely differentiable functions 372

Chapter 7: Representations of the Groups $SU(1,1)$ and $SL(2,\mathbb{R})$ in Mixed Bases. The Hypergeometric Function 374

7.1. The Realization of Representations T_χ in the Space of Functions on the Straight Line 374
7.1.1. Parametrization of $SL(2,\mathbf{R})$ 374
7.1.2. A new realization of representations T_χ 375
7.1.3. Bases of the space $\mathfrak{T}_\chi$ 376
7.1.4. The representation R_χ 378
7.1.5. Infinitesimal operators of representations R_χ 380
7.2. Calculation of the Kernels of Representations R_χ 381
7.2.1. Calculation of $K^{22}(\lambda,\mu;\chi;h)$ and $K^{22}(\lambda,\mu;\chi;u)$ 381
7.2.2. The kernels $K^{22}(\lambda,\mu;\chi;g)$ for triangular matrices . . . 384
7.3. Functional Relations for the Hypergeometric Function . . 386
7.3.1. Relations between infinitesimal operators and representation operators 386
7.3.2. Recurrence formulas 387
7.3.3. The hypergeometric equation and its solutions 390
7.3.4. Integral representations for the hypergeometric function . 391
7.3.5. Formulas of linear transformations 396
7.3.6. Formulas of quadratic transformations 397
7.4. Special Functions Connected with the Hypergeometric Function 398
7.4.1. The functions $\mathfrak{P}_{\lambda\mu}^{\tau}(z)$ and $\mathfrak{Q}_{\lambda\mu}^{\tau}(z)$ 398
7.4.2. Expressions for the kernels $K^{22}(\lambda,\mu;\chi;h)$ in terms of the functions $\mathfrak{P}_{\lambda\mu}^{\tau}(z)$ and $\mathfrak{Q}_{\lambda\mu}^{\tau}(z)$ 401
7.4.3. Jacobi functions 401
7.4.4. Associated Legendre functions of the first and of the second kinds 404
7.4.5. Legendre functions of the second kind 407
7.4.6. Gegenbauer functions 408
7.5. The Mellin Transform and Addition Formulas for the Hypergeometric Function 409
7.5.1. The Mellin transform 409
7.5.2. Mellin transform (degenerate cases) 413
7.5.3. Addition theorems 416
7.5.4. Degenerate cases of addition theorems 419

7.5.5. Integral transforms, connected with the hypergeometric function 422
7.6. The Kernels $K^{33}(\lambda, \mu; \chi; g)$ and Hankel Functions 423
7.6.1. The realization of representations $\widehat{T}_\chi$, connected with the subgroup of triangular matrices 423
7.6.2. Calculation of the kernel of $Q_\chi(s)$ 424
7.6.3. The kernels of $Q_\chi(g)$ 426
7.6.4. Mutually reciprocal integral transforms 427
7.6.5. Functional relations for cylindrical functions 428
7.6.6. Addition theorems for $H^{(1)}_{2\tau+1}(x)$ and $K_{2\tau+1}(x)$ 430
7.7. The Kernels $K^{ij}(\lambda, \mu; \chi; g)$, $i \neq j$, and Special Functions . . 434
7.7.1. Integral representations of the kernels $K^{ij}(\lambda, \mu; \chi; g)$ . . . 434
7.7.2. Some properties of the kernels 437
7.7.3. Calculation of the kernels 439
7.7.4. Mutually reciprocal integral transforms 446
7.7.5. The Mellin transform 447
7.7.6. Addition theorems 450
7.7.7. Discrete series representations of the group $SL(2, \mathbf{R})$ . . . 458
7.7.8. Discrete series representations in the oscillator form . . . 465
7.7.9. Discrete series representations and special functions . . . 468
7.7.10. Integral transforms 471
7.7.11. Pollaczek-Meixner polynomials 473
7.8. Harmonic Analysis on the Group $SL(2, \mathbf{R})$ and Integral Transforms 475
7.8.1. Fourier transform of functions on the group $SL(2, \mathbf{R})$ . . . 475
7.8.2. Characters of irreducible representations of $SL(2, \mathbf{R})$ . . . 476
7.8.3. Derivation of the inversion formula 478
7.8.4. Integral transforms, connected with the Fourier transform on $SL(2, \mathbf{R})$ 483
7.8.5. Decomposition of representations 488
7.8.6. The Laplace operator on the group $SL(2, \mathbf{R})$ 489
7.8.7. Eigenfunctions of the Laplace operator on $SL(2, \mathbf{R})$ . . . 492
7.8.8. The inversion formula for the Jacobi transform 493

Chapter 8: Clebsch-Gordan Coefficients, Racah Coefficients, and Special Functions 498

8.1. Clebsch-Gordan Coefficients of the Group $SU(2)$ 498
8.1.1. Realization of the tensor product $T_{\ell_1} \otimes T_{\ell_2}$ in the space of homogeneous polynomials 498
8.1.2. Clebsch-Gordan coefficients (CGC's) of the group $SU(2)$. 500
8.1.3. Calculation of CGC's 501

8.1.4. Expression for matrix elements of the representation T_ℓ in terms of CGC's 503
8.2. Properties of CGC's of the Group $SU(2)$ 504
8.2.1. Generating functions 504
8.2.2. Symmetry relations 505
8.2.3. Relations between CGC's and matrix elements of irreducible representations 506
8.2.4. Integral representations 507
8.2.5. Expressions for CGC's in the form of finite sums 509
8.2.6. Special values of CGC's 511
8.2.7. Recurrence relations and difference equations for CGC's . 513
8.2.8. Addition theorems 516
8.3. CGC's, the Hypergeometric Function ${}_3F_2(\ldots;1)$ and Jacobi Polynomials 517
8.3.1. CGC's and the hypergeometric function ${}_3F_2(\ldots;1)$ . . . 517
8.3.2. Symmetry relations for the hypergeometric function ${}_3F_2(\ldots;1)$ 520
8.3.3. Summation formulas for hypergeometric series 521
8.3.4. Asymptoptic formulas 523
8.3.5. Recurrence relations and difference equations for the function ${}_3F_2(\ldots;1)$ 524
8.3.6. CGC's and products of Jacobi polynomials 526
8.3.7. CGC's and new recurrence formulas for the functions $P^{\ell}_{mn}(z)$ 527
8.3.8. The Burchnall-Chaundy formula 527
8.4. Racah Coefficients of $SU(2)$ and the Hypergeometric Function ${}_4F_3(\ldots;1)$ 529
8.4.1. Definition of Racah coefficients (RC's) 529
8.4.2. Connection between RC's and CGC's 532
8.4.3. Symmetry relations for RC's 533
8.4.4. RC's and the hypergeometric function ${}_4F_3(\ldots;1)$ 534
8.4.5. Special cases of RC's 536
8.4.6. Expressions for RC's in terms of characters of representations 537
8.4.7. The addition theorem for RC's 538
8.4.8. The Biedenharn-Elliott identity 539
8.4.9. Recurrence relations for RC's 540
8.4.10. CGC as a limit of RC 541
8.4.11. New addition theorems for CGC's 542
8.4.12. Symmetry relations for ${}_4F_3(\ldots;1)$ 543
8.4.13. The Whipple formula and its consequences 543

8.5. Hahn and Racah Polynomials 546
8.5.1. CGC's and Hahn polynomials 546
8.5.2. Dual Hahn polynomials (Eberlane polynomials) 549
8.5.3. The multiplication formula and the addition theorem for Krawtchouk polynomials 550
8.5.4. Racah polynomials 552
8.5.5. Wilson polynomials 556
8.5.6. Addition Theorems for Hahn and Racah polynomials . . 558
8.5.7. Hahn polynomials as a limit of Racah and Wilson polynomials 559
8.6. Clebsch-Gordan and Racah Coefficients of the Group S and Orthogonal Polynomials 561
8.6.1. The tensor product of representations of the group S . . 561
8.6.2. Clebsch-Gordan coefficients 563
8.6.3. The generating function and symmetry relations 566
8.6.4. Clebsch-Gordan coefficients and Laguerre polynomials . . 567
8.6.5. The multiplication formula and the addition theorem for Charlier polynomials 568
8.6.6. Racah coefficients (RC's) 569
8.7. Clebsch-Gordan Coefficients of the Group $SL(2,\mathbf{R})$ 572
8.7.1. CGC's of the tensor product of infinite and finite dimensional representations 572
8.7.2. Expansion of products of functions $\mathfrak{P}^{\tau}_{mn}(z)$ and $P^{\ell}_{mn}(z)$. 576
8.7.3. Recurrence relations for $\mathfrak{P}^{\tau}_{mn}(\cosh t)$ 577
8.7.4. The tensor product of discrete series representations . . . 578
8.7.5. CGC's for continuous bases 580
8.7.6. CGC's for discrete series representations and special functions 583
8.7.7. Other tensor products 587
8.7.8. CGC's for the tensor product $\widehat{T}_{\chi_1} \otimes \widehat{T}_{\chi_2}$ 588
8.7.9. CGC's for the principal unitary series representations . . 590
Bibliography 595
Subject Index 599