

## Table of Contents

### **Series Editor's Preface**

<b>List of Special Symbols</b>	xvii
<b>Chapter 14:</b>	
<b>Quantum Groups, <math>q</math>-Orthogonal Polynomials and Basic Hypergeometric Functions</b>	1
<b>14.1. <math>q</math>-Analysis and Basic Hypergeometric Functions</b>	1
14.1.1. $q$ -Factorials	1
14.1.2. The function ${}_1\varphi_0(a; q, x)$	3
14.1.3. Expressions for $(a; q)_n$ , $(a; q)_n^{-1}$ and their corollaries	5
14.1.4. $q$ -Analog of the binomial formula	8
14.1.5. $q$ -Differentiation and $q$ -integration	9
14.1.6. $q$ -Analogs of the exponential and of the trigonometrical functions	12
14.1.7. $q$ -Analogs of the gamma- and beta-functions	14
14.1.8. Properties of the basic hypergeometric function ${}_2\varphi_1$	17
<b>14.2. Hopf Algebras, Their Representations and Corepresentations</b>	20
14.2.1. Introduction	20
14.2.2. Algebra of functions on a group	21
14.2.3. Definition of a Hopf algebra	23
14.2.4. Coordinate functions	25
14.2.5. Representations and corepresentations of Hopf algebras	26
<b>14.3. Representations of the Quantum Algebra <math>U_q(\mathfrak{sl}_2)</math> and Its Clebsch-Gordan Coefficients</b>	29
14.3.1. The quantum algebra $U_q(\mathfrak{sl}_2)$ and its real forms	29
14.3.2. Finite dimensional representations of $U_q(\mathfrak{sl}_2)$	32
14.3.3. The tensor product of representations	34
14.3.4. Calculation of CGC's	36
14.3.5. Expressions for CGC's in terms of the function ${}_3\Phi_2$	38
14.3.6. Special cases of CGC's	39
14.3.7. Symmetries of CGC's	40
14.3.8. Generating functions for CGC's	41
14.3.9. The difference equation for CGC's	45
14.3.10. Recurrence relations for CGC's	45
<b>14.4. Matrix Elements of Representations of <math>U_q(\mathfrak{sl}_2)</math></b>	47

<b>14.4.1.</b>	Introduction . . . . .	47
<b>14.4.2.</b>	Relations for $\pi_{ij}$ . . . . .	48
<b>14.4.3.</b>	Calculation of matrix elements . . . . .	49
<b>14.4.4.</b>	Expressions in terms of $q$ -Jacobi polynomials . . . . .	51
<b>14.5.</b>	<b>Racah Coefficients of the Algebra <math>U_q(\mathfrak{sl}_2)</math></b> . . . . .	52
<b>14.5.1.</b>	Properties of Racah coefficients . . . . .	52
<b>14.5.2.</b>	Calculation of RC's . . . . .	53
<b>14.5.3.</b>	Special values of RC's . . . . .	56
<b>14.5.4.</b>	The Biedenharn-Elliott identity . . . . .	57
<b>14.5.5.</b>	The addition theorem for RC's . . . . .	58
<b>14.5.6.</b>	Generalization of the Biedenharn-Elliott identity . . . . .	59
<b>14.5.7.</b>	CGC's as a limit of RC's . . . . .	60
<b>14.5.8.</b>	Other asymptotic formulas for RC's . . . . .	61
<b>14.5.9.</b>	Recurrence relations and the second order difference equation . . . . .	63
<b>14.6.</b>	<b>Representations of the Quantum Algebra <math>U_q(\mathfrak{sl}_2)</math> and <math>q</math>-Orthogonal Polynomials</b> . . . . .	64
<b>14.6.1.</b>	Matrix elements of representations and $q$ -Krawtchouk polynomials . . . . .	64
<b>14.6.2.</b>	The product and the addition theorems for $q$ -Krawtchouk polynomials . . . . .	65
<b>14.6.3.</b>	A $q$ -analog of the Burchnall-Chaundy formula . . . . .	66
<b>14.6.4.</b>	CGC's and $q$ -Hahn polynomials . . . . .	68
<b>14.6.5.</b>	RC's and $q$ -Racah polynomials . . . . .	70
<b>14.6.6.</b>	The addition theorem for $q$ -Hahn polynomials . . . . .	72
<b>14.6.7.</b>	The addition formula for $q$ -Racah polynomials . . . . .	76
<b>14.6.8.</b>	RC's and properties of basic hypergeometric functions . . . . .	77
<b>14.6.9.</b>	Relations for little $q$ -Jacobi polynomials and CGC's . . . . .	79
<b>14.7.</b>	<b><math>q</math>-Askey-Wilson Polynomials and their Special Cases</b> . . . . .	81
<b>14.7.1.</b>	$q$ -Askey-Wilson polynomials . . . . .	81
<b>14.7.2.</b>	Properties of $q$ -Askey-Wilson polynomials . . . . .	85
<b>14.7.3.</b>	$q$ -Gegenbauer polynomials . . . . .	87
<b>14.7.4.</b>	Continuous $q$ -Hermite polynomials . . . . .	90
<b>14.7.5.</b>	Continuous $q$ -Jacobi polynomials . . . . .	91
<b>14.7.6.</b>	Big $q$ -Jacobi polynomials . . . . .	94
<b>14.8.</b>	<b>Analysis on the Quantum Group <math>SL_q(2, \mathbb{C})</math> and Little <math>q</math>-Jacobi Polynomials</b> . . . . .	97
<b>14.8.1.</b>	The algebra of functions on the quantum group $SL_q(2, \mathbb{C})$ . . . . .	97
<b>14.8.2.</b>	Decomposition of the Hopf algebra $A$ . . . . .	99

<b>14.8.3.</b>	Finite dimensional corepresentations of $A$	101
<b>14.8.4.</b>	Calculation of matrix elements	103
<b>14.8.5.</b>	Irreducibility of the representations $T_\ell$	105
<b>14.8.6.</b>	Invariant integral on $A$	106
<b>14.8.7.</b>	Scalar products on $A(SU_q(2))$	108
<b>14.8.8.</b>	Unitary representations of the quantum group $SU_q(2)$	110
<b>14.8.9.</b>	An analog of the Peter-Weyl theorem	112
<b>14.8.10.</b>	The Fourier transform on the quantum group $SU_q(2)$	113
<b>14.8.11.</b>	Orthogonality of little $q$ -Jacobi polynomials. $q$ -Legendre and Wall polynomials	114
<b>14.8.12.</b>	Addition and product formulas for $q$ -Legendre polynomials	116
<b>14.8.13.</b>	The differential form of the quantum group $SL_q(2, \mathbb{C})$	119
<b>14.8.14.</b>	The differential form of corepresentations	120
<b>14.8.15.</b>	The difference equation for little $q$ -Jacobi polynomials	122
<b>14.8.16.</b>	The Rodrigues formula for little $q$ -Jacobi polynomials	123
<b>14.9.</b>	<b>Representations of the Quantum Group <math>SU_q(2)</math> on Quantum Spheres and <math>q</math>-Orthogonal Polynomials</b>	124
<b>14.9.1.</b>	The algebra of functions on a quantum 2-sphere	124
<b>14.9.2.</b>	Decomposition of the algebra $A(S_q^2)$	126
<b>14.9.3.</b>	An invariant integral in $S_q^2$	127
<b>14.9.4.</b>	Spherical functions on $A(S_q^2)$	128
<b>14.9.5.</b>	The orthogonality relation	130
<b>14.9.6.</b>	The difference equation	131
<b>14.9.7.</b>	The algebra of functions on a quantum 3-sphere	132
<b>14.9.8.</b>	Spherical functions on $S_q^3$ and big $q$ -Jacobi polynomials	135

**Chapter 15:****Semisimple Lie Groups and Related  
Homogeneous Spaces**

<b>15.1.</b>	<b>Decompositions of Semisimple Lie Algebras and Groups</b>	137
<b>15.1.1.</b>	Decompositions of $\mathfrak{sl}(n, \mathbb{C})$ and $SL(n, \mathbb{C})$	137
<b>15.1.2.</b>	Cartan subgroups and subalgebras. Roots and root subspaces	139
<b>15.1.3.</b>	Generating elements of complex semisimple Lie algebras	150
<b>15.1.4.</b>	Restricted roots and root subspaces	152
<b>15.1.5.</b>	Real simple Lie groups and algebras	153
<b>15.1.6.</b>	The Iwasawa decomposition	154
<b>15.1.7.</b>	The Gauss decomposition	158

<b>15.1.8.</b>	The Bruhat decomposition . . . . .	159
<b>15.1.9.</b>	The Cartan decomposition . . . . .	161
<b>15.1.10.</b>	Decompositions of classical groups . . . . .	164
<b>15.1.11.</b>	Noncompact analogues of the Iwasawa and Cartan decompositions . . . . .	167
<b>15.1.12.</b>	Block (partial) decompositions of groups and parabolic subgroups . . . . .	168
<b>15.1.13.</b>	Limits and contractions of Lie algebras . . . . .	169
<b>15.2.</b>	<b>Homogeneous Spaces with Semisimple Motion Groups</b> . .	172
<b>15.2.1.</b>	Homogeneous self-adjoint cones . . . . .	172
<b>15.2.2.</b>	Hermitian symmetric space . . . . .	174
<b>15.2.3.</b>	Tube domains . . . . .	179
<b>15.2.4.</b>	Parametrizations of the space $\mathfrak{P}_m(\mathbb{F})$ . . . . .	181
<b>15.2.5.</b>	Spherical, conic and flag spaces . . . . .	183
<b>15.3.</b>	<b>Invariant Metrics, Measures, and Differential Operators on Lie Groups and on Homogeneous Spaces</b> . . . . .	185
<b>15.3.1.</b>	Relations between invariant measures on Lie groups . .	185
<b>15.3.2.</b>	Invariant metrics and measures on homogeneous cones .	190
<b>15.3.3.</b>	Laplace operators on semisimple Lie groups and their radial parts . . . . .	194

## Chapter 16:

### Representations of Semisimple Lie Groups and

### Their Matrix Elements

199

<b>16.1.</b>	<b>Irreducible Finite Dimensional Representations of Lie Groups</b> . . . . .	199
<b>16.1.1.</b>	Representations of Lie groups with normal Gauss decompositions . . . . .	199
<b>16.1.2.</b>	Finite dimensional irreducible representations of classical complex Lie groups . . . . .	201
<b>16.1.3.</b>	Block Gauss decompositions and representations . . . .	204
<b>16.1.4.</b>	The Kostant theorem on separation of variables . . . .	205
<b>16.1.5.</b>	Realization of finite dimensional representations on spaces of polynomials in minors . . . . .	208
<b>16.1.6.</b>	Decomposition of symmetric powers of finite dimensional irreducible representations . . . . .	211
<b>16.1.7.</b>	Restrictions of irreducible representations of classical groups . . . . .	214
<b>16.1.8.</b>	The scalar product in the space $\mathcal{P}(\mathfrak{M}_{mn}(\mathbb{F}))$ . . . .	216

<b>16.2. The Principal Series Representations of Classical Lie Groups and Their Matrix Elements . . . . .</b>	217
<b>16.2.1. The principal series representations of the group <math>GL(n, \mathbb{C})</math> . . . . .</b>	217
<b>16.2.2. Representations of real semisimple Lie groups . . . . .</b>	220
<b>16.2.3. Realization of the principal series representations in spaces of functions on matrix cones and hyperboloids . . . . .</b>	225
<b>16.2.4. Relations between finite and infinite dimensional representations of classical groups . . . . .</b>	228
<b>16.2.5. Representations of nilpotent groups and of semidirect products . . . . .</b>	229
<b>16.2.6. Block splittings of matrices of irreducible representations . . . . .</b>	232
<b>16.2.7. The orthogonality relation for rows and columns . . . . .</b>	235
<b>16.2.8. Block matrix elements of irreducible representations of semisimple Lie groups . . . . .</b>	238
<b>16.2.9. Integral expression for matrix elements of the principal series representations . . . . .</b>	239
<b>16.3. Hypergeometric Functions of Many Variables and Representations of the Group <math>GL(n, \mathbb{R})</math> . . . . .</b>	240
<b>16.3.1. The Lauricella functions . . . . .</b>	240
<b>16.3.2. The most degenerate series representations of the group <math>SL(n, \mathbb{R})</math> . . . . .</b>	244
<b>16.3.3. Generalized beta-functions and the kernels <math>K_{AB}^\lambda</math> . . . . .</b>	245
<b>16.3.4. The Lauricella functions and the kernels <math>K_{AB}^\gamma</math> . . . . .</b>	247
<b>Chapter 17: Group Representations and Special Functions of a Matrix Argument . . . . .</b>	251
<b>17.1. Elementary Functions of a Matrix Argument. Gamma-Function and Beta-Function . . . . .</b>	251
<b>17.1.1. Elementary functions of a matrix argument . . . . .</b>	251
<b>17.1.2. The Fourier and the Laplace transforms of functions of a matrix argument . . . . .</b>	252
<b>17.1.3. The Fourier transform of harmonic polynomials . . . . .</b>	255
<b>17.1.4. Generalized gamma-functions . . . . .</b>	257
<b>17.1.5. Generalized beta-functions . . . . .</b>	260
<b>17.1.6. Matrix analogues of the integral <math>\int\limits_{-\infty}^{\infty} (1+x^2)^{-\alpha} dx</math> . . . . .</b>	262

17.1.7.	Matrix analogues of the integral $\int_0^1 (1-x^2)^\lambda dx$	265
<b>17.2.</b>	<b>Zonal Spherical Functions and Characters</b>	270
17.2.1.	Gel'fand pairs	270
17.2.2.	Zonal spherical functions and their properties	272
17.2.3.	Characters of representations as spherical functions	275
17.2.4.	Evaluation of characters of irreducible representations of classical Lie groups	276
17.2.5.	Identities for characters of irreducible representations of $GL(n, \mathbb{C})$	280
17.2.6.	Evaluation of zonal spherical functions of classical complex Lie groups	285
17.2.7.	The Green functions	286
17.2.8.	Spherical transforms	288
17.2.9.	Average values and Laplace operators	290
17.2.10.	The algebra of representations	291
<b>17.3.</b>	<b>Zonal and Intertwining Polynomials</b>	295
17.3.1.	Recurrence formulas	295
17.3.2.	Spherical functions as orthogonal polynomials	298
17.3.3.	Invariant polynomials	300
17.3.4.	Zonal spherical polynomials and their properties	302
17.3.5.	Integral representations of zonal spherical polynomials	305
17.3.6.	The Laplace transform of zonal polynomials	305
17.3.7.	Evaluation of zonal spherical polynomials	307
17.3.8.	Intertwining functions	309
17.3.9.	Generalized Jacobi polynomials	313
17.3.10.	Generalized Jacobi polynomials and intertwining operators	317
17.3.11.	Zonal spherical functions and generalized Jacobi and Bessel functions	318
17.3.12.	Generalized Gel'fand pairs	321
17.3.13.	Ordered symmetric spaces and Volterra algebras	322
17.3.14.	Zonal spherical functions on the space $\mathfrak{P}_{pq}(\mathbf{F})$	324
<b>17.4.</b>	<b>Hypergeometric Functions of a Matrix Argument</b>	328
17.4.1.	Hypergeometric functions on $\mathfrak{H}_m(\mathbf{F})$	328
17.4.2.	Bessel functions of a matrix argument	331
17.4.3.	Hankel transforms of functions of a matrix argument	333
17.4.4.	Bessel functions of the second kind in a matrix argument	340
17.4.5.	Macdonald functions of a matrix argument	343

<b>17.4.6.</b>	The confluent hypergeometric function of a matrix argument . . . . .	346
<b>17.4.7.</b>	Whittaker functions of a matrix argument . . . . .	348
<b>17.4.8.</b>	Generalized Laguerre polynomials . . . . .	351
<b>17.4.9.</b>	The Gauss hypergeometric function of a matrix argument	354
<b>17.4.10.</b>	Jacobi and Gegenbauer functions of a matrix argument . . . . .	357
<b>Chapter 18:</b>		
<b>Representations in the Gel'fand-Tsetlin Basis and Special Functions</b>		361
<b>18.1.</b>	<b>Infinitesimal Operators of Representations of the Groups <math>U(n)</math> and <math>SO(n)</math></b> . . . . .	361
<b>18.1.1.</b>	The Gel'fand-Tsetlin basis . . . . .	361
<b>18.1.2.</b>	Infinitesimal operators of irreducible representations . . . . .	362
<b>18.2.</b>	<b>Clebsch-Gordan Coefficients for the Gel'fand-Tsetlin Basis</b> . . . . .	365
<b>18.2.1.</b>	Definition of Clebsch-Gordan coefficients . . . . .	365
<b>18.2.2.</b>	Scalar factors . . . . .	367
<b>18.2.3.</b>	Tensor operators . . . . .	370
<b>18.2.4.</b>	The Wigner-Eckart theorem . . . . .	371
<b>18.2.5.</b>	Matrix elements of the operators $E_{n-1,n}^k$ and $E_{n,n-1}^k$ of representations of $\mathfrak{gl}(n, \mathbb{C})$ . . . . .	373
<b>18.2.6.</b>	CGC's for the tensor product $T_{\mathbf{m}_n} \otimes T_{(p,0)}$ . . . . .	376
<b>18.2.7.</b>	Evaluation of scalar factors . . . . .	378
<b>18.2.8.</b>	CGC's of the tensor product $T_{\mathbf{m}} \otimes T_{(0,-p)}$ . . . . .	381
<b>18.2.9.</b>	CGC's of the tensor product of $T_{\mathbf{m}_n}$ with fundamental representations . . . . .	382
<b>18.2.10.</b>	CGC's of the tensor product $T_{\mathbf{m}_n} \otimes T_{(1,0)}$ . . . . .	384
<b>18.3.</b>	<b>Matrix Elements of Representations of the Group <math>GL(n, \mathbb{C})</math> and General Beta-Functions</b> . . . . .	388
<b>18.3.1.</b>	Matrix elements of irreducible finite dimensional representations of $GL(n, \mathbb{C})$ . . . . .	388
<b>18.3.2.</b>	General beta-functions, related to the Gel'fand-Tsetlin basis . . . . .	389
<b>18.3.3.</b>	Matrix beta-functions . . . . .	391
<b>18.3.4.</b>	Recurrence formulas for general beta-functions . . . . .	393
<b>18.4.</b>	<b>Representations of <math>U(n)</math> in the Gel'fand-Tsetlin Bases and Special Functions</b> . . . . .	395
<b>18.4.1.</b>	Matrix elements of the representations of the group $U(n)$	395

<b>18.4.2.</b>	The symmetry relations . . . . .	397
<b>18.4.3.</b>	Matrix elements of the fundamental representations . . . . .	400
<b>18.4.4.</b>	Matrix elements and CGC's . . . . .	400
<b>18.4.5.</b>	Matrix elements of representations of $U(n)$ and generalizations of classical polynomials of a discrete variable . . . . .	403
<b>18.4.6.</b>	Representations of $U(n)$ and generalized Jacobi polynomials . . . . .	403
<b>18.4.7.</b>	The addition theorem for the polynomials $F_1$ and $F_2$ . . . . .	406
<b>18.4.8.</b>	Recurrence relations . . . . .	406
<b>18.4.9.</b>	Orthogonality relations . . . . .	408
<b>18.5.</b>	<b>Matrix Elements of Representations of the Groups</b>	
	<i><math>U(n - 1, 1)</math>, <math>IU(n - 1)</math> in the Gel'fand-Tsetlin Basis</i> . . . . .	409
<b>18.5.1.</b>	Representations of the group $U(n - 1, 1)$ . . . . .	409
<b>18.5.2.</b>	Matrix elements of the representations of $U(n - 1, 1)$ . . . . .	411
<b>18.5.3.</b>	Representations of the group $IU(n - 1)$ . . . . .	413
<b>18.5.4.</b>	Matrix elements of the representations of $IU(n - 1)$ . . . . .	414
<b>18.6.</b>	<b>Representations of the Groups <math>SO(n)</math>, <math>SO_0(n - 1, 1)</math>, <math>ISO(n - 1)</math> and Special Functions with Matrix Indices</b> . . . . .	416
<b>18.6.1.</b>	Introduction . . . . .	416
<b>18.6.2.</b>	Representations of the groups $SO_0(n - 1, 1)$ and $ISO(n - 1)$ . . . . .	417
<b>18.6.3.</b>	Matroms of representations . . . . .	421
<b>18.6.4.</b>	Differential relations of the first order . . . . .	422
<b>18.6.5.</b>	Differential equations of the second order . . . . .	425
<b>18.6.6.</b>	Bessel and Jacobi functions with matrix indices . . . . .	427
<b>18.7</b>	<b>Orthogonal Polynomials of Many Discrete and Continuous Variables</b> . . . . .	432
<b>18.7.1.</b>	General procedure of spectral analysis of infinitesimal operators and matroms . . . . .	432
<b>18.7.2.</b>	Partial difference equations connected with infinitesimal operators and matroms . . . . .	433
<b>18.7.3.</b>	Spectral characteristics of discrete equations . . . . .	438
<b>18.7.4.</b>	Continuous analogs of discrete polynomials . . . . .	440
<b>18.7.5.</b>	Expansions of representation matrix elements . . . . .	444
<b>Chapter 19:</b>		
<b>Modular Forms, Theta Functions and Representations of Affine Lie Algebras</b>		447
<b>19.1.</b>	<b>Modular Forms</b> . . . . .	447

<b>19.1.1.</b>	Linear-fractional transformations of the upper half-plane	447
<b>19.1.2.</b>	The transformation group $SL(2, \mathbb{Z})$	449
<b>19.1.3.</b>	Congruence subgroups of $SL(2, \mathbb{Z})$	451
<b>19.1.4.</b>	Modular forms of an integral weight	453
<b>19.1.5.</b>	Eisenstein series	456
<b>19.1.6.</b>	Modular forms with multiplier system	459
<b>19.2.</b>	<b>Theta Functions</b>	462
<b>19.2.1.</b>	The Jacobi theta functions	462
<b>19.2.2.</b>	Functional equation	465
<b>19.2.3.</b>	The Jacobi theta functions and the heat equation	470
<b>19.2.4.</b>	Factorization of the theta function into infinite product	471
<b>19.2.5.</b>	Theta functions with characteristics	473
<b>19.2.6.</b>	Theta functions of many variables	476
<b>19.2.7.</b>	The symplectic group	479
<b>19.2.8.</b>	The functional equation for the theta function of many variables	480
<b>19.2.9.</b>	Theta functions of many variables with characteristics	483
<b>19.2.10.</b>	Relations for products of theta functions	484
<b>19.3.</b>	<b>Theta Functions and the Decomposition of Quasi-Regular Representation of the Heisenberg Group on the Cube</b>	486
<b>19.3.1.</b>	Auxiliary theta functions	486
<b>19.3.2.</b>	The space $H_\Omega(\mathbf{a}/n)$	491
<b>19.3.3.</b>	Decomposition of the quasi-regular representation	494
<b>19.3.4.</b>	The orthonormal basis of the space $H_\Omega(\mathbf{a}/n)$	497
<b>19.4.</b>	<b>Affine Lie Algebras</b>	498
<b>19.4.1.</b>	Non-twisted affine Lie algebras	498
<b>19.4.2.</b>	Roots and root elements of non-twisted affine Lie algebras	500
<b>19.4.3.</b>	The Virasoro algebra	505
<b>19.4.4.</b>	The affine Lie algebra $A_1^{(1)}$	506
<b>19.4.5.</b>	Twisted affine Lie algebras	509
<b>19.4.6.</b>	The affine Lie algebra $A_2^{(2)}$	515
<b>19.4.7.</b>	Classification of affine Lie algebras	519
<b>19.4.8.</b>	The universal enveloping algebra	524
<b>19.5.</b>	<b>Representations of Affine Lie Algebras and their Characters</b>	526
<b>19.5.1.</b>	Integrable and weight representations	526
<b>19.5.2.</b>	Verma modules	527
<b>19.5.3.</b>	Integrable representations with highest weight	529

<b>19.5.4.</b> Characters of integrable representations . . . . .	530
<b>19.6. Characters of Representations of the Affine Lie Algebras and Combinatorial Identities . . . . .</b>	534
<b>19.6.1.</b> The denominator formula for the algebras $A_1^{(1)}$ , $A_2^{(2)}$ and the Jacobi identity . . . . .	534
<b>19.6.2.</b> Specialized characters of the algebra $A_1^{(1)}$ . . . . .	536
<b>19.6.3.</b> Specialized characters of the algebra $A_2^{(2)}$ . . . . .	540
<b>19.7. Characters of Representations and Theta Functions . . . . .</b>	542
<b>19.7.1.</b> An other form of theta functions . . . . .	542
<b>19.7.2.</b> The lattices $M$ and $M'$ . . . . .	546
<b>19.7.3.</b> Maximal weights of irreducible integrable representations	552
<b>19.7.4.</b> Characters of representations and theta functions . . .	553
<b>19.7.5.</b> The functions $A_\lambda$ and $A'_\lambda$ . . . . .	557
<b>19.7.6.</b> Expressions for the function $A_\rho$ . . . . .	561
<b>19.7.7.</b> Transformation properties of the function $A_\rho$ . . . .	563
<b>19.7.8.</b> Polynomial algebras . . . . .	568
<b>19.8. The String Function . . . . .</b>	569
<b>19.8.1.</b> Properties of the string function . . . . .	569
<b>19.8.2.</b> The matrix of the string functions . . . . .	572
<b>19.8.3.</b> Explicit expressions for the string functions . . . .	575
<b>19.8.4.</b> Formulas for the partition function . . . . .	579
<b>19.8.5.</b> Hecke modular forms and the string function for $A_1^{(1)}$ .	582
<b>19.8.6.</b> Applications of the string functions . . . . .	588
<b>19.9. Reduction of Representations of an Affine Lie Algebra onto a Subalgebra and Hecke Modular Forms . . . . .</b>	590
<b>19.9.1.</b> The functions $E_{jk}^\ell$ . . . . .	590
<b>19.9.2.</b> The matrix $(e_{jk}^\ell)$ . . . . .	593
<b>19.9.3.</b> Evaluation of $E_{jk}^\ell$ . . . . .	594
<b>19.9.4.</b> Reduction $C_{2\ell}^{(1)} \supset C_\ell^{(1)}$ . . . . .	600
<b>Bibliography . . . . .</b>	603
<b>Bibliography Notes . . . . .</b>	621
<b>Subject Index . . . . .</b>	625