

Contents

Introduction to the First Edition	xi
Introduction to the Second Edition	xvii
Chapter 0. Notation and Preliminaries	1
1. Notation	1
2. Representations of Lie groups	2
3. Linear algebraic and reductive groups	4
Chapter I. Relative Lie Algebra Cohomology	7
1. Lie algebra cohomology	7
2. The Ext functors for $(\mathfrak{g}, \mathfrak{k})$ -modules	9
3. Long exact sequences and Ext	13
4. A vanishing theorem	15
5. Extension to (\mathfrak{g}, K) -modules	16
6. $(\mathfrak{g}, \mathfrak{k}, L)$ -modules. A Hochschild-Serre spectral sequence in the relative case	19
7. Poincaré duality	22
8. The Zuckerman functors	25
Chapter II. Scalar Product, Laplacian and Casimir Element	31
1. Notation and general remarks	31
2. Scalar product	33
3. Special cases	36
4. The bigrading in the bounded symmetric domain case	37
5. Cohomology with respect to square integrable representations	40
6. Spinors and the spin Laplacian	43
7. Vanishing theorems using spinors	47
8. Matsushima's vanishing theorem	50
9. Direct products	54
10. Sharp vanishing theorems	55
Chapter III. Cohomology with Respect to an Induced Representation	59
1. Notation and conventions	59
2. Induced representations and their K -finite vectors	61
3. Cohomology with respect to principal series representations	64
4. Fundamental parabolic subgroups	66
5. Tempered representations	69
6. Representations induced from tempered ones	70
7. Appendix: C^∞ vectors in certain induced representations	70

Chapter IV. The Langlands Classification and Uniformly Bounded Representations	75
1. Some results of Harish-Chandra	75
2. Some ideas of Casselman	78
3. The Langlands classification (first step)	81
4. The Langlands classification (second step)	84
5. A necessary condition for uniform boundedness	87
6. Appendix: Langlands' geometric lemmas	91
7. Appendix: A lemma on exponential polynomial series	94
Chapter V. Cohomology with Coefficients in $\Pi_\infty(G)$	97
1. Preliminaries	97
2. The class $\Pi_\infty(G)$	100
3. A vanishing theorem for the class $\Pi_\infty(G)$	100
4. Cohomology with coefficients in the Steinberg representation	103
5. H^1 and the topology of $\mathcal{E}(G)$	107
6. A more detailed examination of first cohomology	110
Chapter VI. The Computation of Certain Cohomology Groups	115
0. Translation functors	115
1. Cohomology with respect to minimal non-tempered representations. I	117
2. Cohomology with respect to minimal non-tempered representations. II	120
3. Semi-simple Lie groups with \mathbf{R} -rank 1	122
4. The groups $\mathbf{SO}(n, 1)$ and $\mathbf{SU}(n, 1)$	127
5. The Vogan-Zuckerman theorem	134
Chapter VII. Cohomology of Discrete Subgroups and Lie Algebra Cohomology	137
1. Manifolds	137
2. Discrete subgroups	139
3. Γ cocompact, E a unitary Γ -module	142
4. G semi-simple, Γ cocompact, E a unitary Γ -module	145
5. Γ cocompact, E a G -module	147
6. G semi-simple, Γ cocompact, E a G -module	149
Chapter VIII. The Construction of Certain Unitary Representations and the Computation of the Corresponding Cohomology Groups	151
1. The oscillator representation	151
2. The decomposition of the restriction of the oscillator representation to certain subgroups	155
3. The theta distributions	161
4. The reciprocity formula	164
5. The imbedding of V_i into $L^2(\Gamma \backslash G)$	165
Chapter IX. Continuous Cohomology and Differentiable Cohomology	169
Introduction	169
1. Continuous cohomology for locally compact groups	170
2. Shapiro's lemma	175

3. Hausdorff cohomology	177
4. Spectral sequences	178
5. Differentiable cohomology and continuous cohomology for Lie groups	180
6. Further results on differentiable cohomology	184
Chapter X. Continuous and Differentiable Cohomology for Locally Compact Totally Disconnected Groups	191
1. Continuous and smooth cohomology	191
2. Cohomology of reductive groups and buildings	196
3. Representations of reductive groups	199
4. Cohomology with respect to irreducible admissible representations	200
5. Forgetting the topology	205
6. Cohomology of products	207
Chapter XI. Cohomology with Coefficients in $\Pi_\infty(G)$: The p -adic Case	211
1. Some results of Harish-Chandra	211
2. The Langlands classification (p -adic case)	215
3. Uniformly bounded representations and $\Pi_\infty(G)$	218
4. Another proof of the non-unitarizability of the V_J 's	221
Chapter XII. Differentiable Cohomology for Products of Real Lie Groups and T.D. Groups	225
0. Homological algebra over idempotented algebras	225
1. Differentiable cohomology	226
2. Modules of K -finite vectors	228
3. Cohomology of products	230
Chapter XIII. Cohomology of Discrete Cocompact Subgroups	233
1. Subgroups of products of Lie groups and t.d. groups	233
2. Products of reductive groups	236
3. Irreducible subgroups of semi-simple groups	239
4. The Γ -module E is the restriction of a rational G -module	243
Chapter XIV. Non-cocompact S -arithmetic Subgroups	247
1. General properties	247
2. Stable cohomology	247
3. The use of L^2 cohomology	249
4. S -arithmetic subgroups	251
Bibliography	253
Index	259