

# CONTENTS

INTRODUCTION . . . . .	9
CHAPTER 1. RECURSIVE FUNCTIONS . . . . .	15
1. Constructive objects . . . . .	15
2. Canonical systems . . . . .	17
3. Recursively enumerable sets and relations . . . . .	18
4. The equation calculus . . . . .	23
5. Turing machines . . . . .	25
6. Church's thesis . . . . .	28
7. The universal system . . . . .	30
8. Enumeration theorem for partial recursive functions . . . . .	34
9. Iteration theorems . . . . .	36
10. Recursive inseparability . . . . .	37
11. Kleene's fixed point theorem . . . . .	38
CHAPTER 2. ELEMENTARY CONSTRUCTIVE ANALYSIS . . . . .	40
12. Neighbourhoods, approximations and constructive points . . . . .	40
13. Richard's paradox and the non enumerability of the continuum . . . . .	43
14. Computable real numbers . . . . .	45
15. Undecidability results for computable real numbers . . . . .	47
16. Specker's sequence . . . . .	50
17. Open and closed sets . . . . .	51
18. Heine-Borel covering theorem . . . . .	52
19. Located closed sets . . . . .	54
20. Inner and outer limit sets . . . . .	55
21. Baire's category theorem . . . . .	57
22. Partial recursive functionals . . . . .	58
23. Maximal recursive functionals . . . . .	60
24. Two theorems of classical function theory . . . . .	64
CHAPTER 3. ORDINAL NUMBERS AND BOREL SETS . . . . .	68
25. Definition of the ordinals of the second number class . . . . .	68
26. Equality and order relations between ordinal numbers . . . . .	70
27. Non enumerability of the second number class . . . . .	75
28. Open sets in the Baire space . . . . .	75
29. Brouwer's fan theorem . . . . .	77
30. Borel sets . . . . .	79
31. A constructive version of Gödel's completeness theorem . . . . .	84
32. Completeness of second order logic with cut . . . . .	87

<b>CHAPTER 4. MEASURE THEORY . . . . .</b>	<b>91</b>
<b>33. Extension of a measure and its basic properties . . . . .</b>	<b>91</b>
<b>34. Measurable and non measurable open sets. Brouwer's theorem . .</b>	<b>97</b>
<b>35. Sets of measure zero. . . . .</b>	<b>101</b>
<b>REFERENCES . . . . .</b>	<b>104</b>
<b>INDEX . . . . .</b>	<b>107</b>