Contents

Preface
Introduction 1

Part I
Boolean Valued Analysis

Chapter 1 Boolean Valued Analysis Using Projection Algebras 6
§ 1. Hilbert space 6
§ 2. The model $V^{(B)}$ 7
§ 3. Real numbers in $V^{(B)}$ 15
§ 4. The interpretation of elementary theorems 21
§ 5. Miscellaneous interpretations 29
§ 6. Convergence 31
§ 7. Semi-group of self-adjoint operators 33
§ 8. Complete Boolean algebras of Banach spaces 38
§ 9. Piecewise convergence 40
§ 10. Simultaneous spectrums 42
§ 11. Quantum Logic 46

Chapter 2 Boolean Valued Analysis Using Measure Algebras 51
§ 1. Measure algebras 51
§ 2. Real numbers in the model 52
§ 3. Continuous functions in the model 56
§ 4. Baire functions and Borel sets in the model 60
§ 5. Integration and differentiation in the model 63
§ 6. Relation between projection algebras and measure algebras 68

References 71

Part II
A Conservative Extension of Peano Arithmetic

Chapter 1 Real Analysis 77
§ 1. The first system 77

§ 2. The second system 89
§ 3. Defining reals 91
§ 4. Functions 93
§ 5. Sequences 96
§ 6. Continuous functions 98
§ 7. Differentiation 101
§ 8. Integration 104
§ 9. Sequences of functions 106
§ 10. Infinite series and series of functions 107
§ 11. Higher derivatives 110
§ 12. Functions of several variables 112

Chapter 2 Complex Analysis 114
§ 1. A system of complex numbers 114
§ 2. Analytic functions 119
§ 3. Integration 122
§ 4. Cauchy's Theorem for nice regions 125
§ 5. Cauchy's Integral Formula 127
§ 6. The general form of Cauchy's Theorem 132
§ 7. Miscellaneous remarks 133
§ 8. Final remarks 134

References 136
Index 139