

TABLE OF CONTENTS

CHAPTER	PAGE
PREFACE	vii
1. INTRODUCTION	1
1.1 Set Theory and the Foundations of Mathematics	1
1.2 Logic and Notation	3
1.3 Axiom Schema of Abstraction and Russell's Paradox	5
1.4 More Paradoxes	8
1.5 Preview of Axioms	12
2. GENERAL DEVELOPMENTS	14
2.1 Preliminaries: Formulas and Definitions	14
2.2 Axioms of Extensionality and Separation	19
2.3 Intersection, Union, and Difference of Sets	24
2.4 Pairing Axiom and Ordered Pairs	30
2.5 Definition by Abstraction	33
2.6 Sum Axiom and Families of Sets	37
2.7 Power Set Axiom	46
2.8 Cartesian Product of Sets	49
2.9 Axiom of Regularity	53
2.10 Summary of Axioms	56
3. RELATIONS AND FUNCTIONS	57
3.1 Operations on Binary Relations	57
3.2 Ordering Relations	68
3.3 Equivalence Relations and Partitions	80
3.4 Functions	86
4. EQUIPOLLENCE, FINITE SETS, AND CARDINAL NUMBERS	91
4.1 Equipollence	91
4.2 Finite Sets	98
4.3 Cardinal Numbers	109
4.4 Finite Cardinals	121

CHAPTER	PAGE
5. FINITE ORDINALS AND DENUMERABLE SETS	127
5.1 Definition and General Properties of Ordinals	127
5.2 Finite Ordinals and Recursive Definitions	135
5.3 Denumerable Sets	150
6. RATIONAL NUMBERS AND REAL NUMBERS	159
6.1 Introduction	159
6.2 Fractions	161
6.3 Non-negative Rational Numbers	166
6.4 Rational Numbers	170
6.5 Cauchy Sequences of Rational Numbers	174
6.6 Real Numbers	181
6.7 Sets of the Power of the Continuum	189
7. TRANSFINITE INDUCTION AND ORDINAL ARITHMETIC	195
7.1 Transfinite Induction and Definition by Transfinite Recursion	195
7.2 Elements of Ordinal Arithmetic	205
7.3 Cardinal Numbers Again and Alephs	224
7.4 Well-Ordered Sets	230
7.5 Revised Summary of Axioms	237
8. THE AXIOM OF CHOICE	239
8.1 Some Applications of the Axiom of Choice	239
8.2 Equivalentents of the Axiom of Choice	243
8.3 Axioms Which Imply the Axiom of Choice	251
REFERENCES	253
GLOSSARY OF SYMBOLS	257
AUTHOR INDEX	259
SUBJECT INDEX	261