

# Contents

<i>Foreword</i>	<i>page ix</i>
<i>Contributors to Sets for Mathematics</i>	xiii
1 Abstract Sets and Mappings	1
1.1 Sets, Mappings, and Composition	1
1.2 Listings, Properties, and Elements	4
1.3 Surjective and Injective Mappings	8
1.4 Associativity and Categories	10
1.5 Separators and the Empty Set	11
1.6 Generalized Elements	15
1.7 Mappings as Properties	17
1.8 Additional Exercises	23
2 Sums, Monomorphisms, and Parts	26
2.1 Sum as a Universal Property	26
2.2 Monomorphisms and Parts	32
2.3 Inclusion and Membership	34
2.4 Characteristic Functions	38
2.5 Inverse Image of a Part	40
2.6 Additional Exercises	44
3 Finite Inverse Limits	48
3.1 Retractions	48
3.2 Isomorphism and Dedekind Finiteness	54
3.3 Cartesian Products and Graphs	58
3.4 Equalizers	66
3.5 Pullbacks	69
3.6 Inverse Limits	71
3.7 Additional Exercises	75

4	Colimits, Epimorphisms, and the Axiom of Choice	78
4.1	Colimits are Dual to Limits	78
4.2	Epimorphisms and Split Surjections	80
4.3	The Axiom of Choice	84
4.4	Partitions and Equivalence Relations	85
4.5	Split Images	89
4.6	The Axiom of Choice as the Distinguishing Property of Constant/Random Sets	92
4.7	Additional Exercises	94
5	Mapping Sets and Exponentials	96
5.1	Natural Bijection and Functoriality	96
5.2	Exponentiation	98
5.3	Functoriality of Function Spaces	102
5.4	Additional Exercises	108
6	Summary of the Axioms and an Example of Variable Sets	111
6.1	Axioms for Abstract Sets and Mappings	111
6.2	Truth Values for Two-Stage Variable Sets	114
6.3	Additional Exercises	117
7	Consequences and Uses of Exponentials	120
7.1	Concrete Duality: The Behavior of Monics and Epics under the Contravariant Functoriality of Exponentiation	120
7.2	The Distributive Law	126
7.3	Cantor's Diagonal Argument	129
7.4	Additional Exercises	134
8	More on Power Sets	136
8.1	Images	136
8.2	The Covariant Power Set Functor	141
8.3	The Natural Map $\mathcal{P}X \longrightarrow 2^{2^X}$	145
8.4	Measuring, Averaging, and Winning with $V$ -Valued Quantities	148
8.5	Additional Exercises	152
9	Introduction to Variable Sets	154
9.1	The Axiom of Infinity: Number Theory	154
9.2	Recursion	157
9.3	Arithmetic of $N$	160
9.4	Additional Exercises	165
10	Models of Additional Variation	167
10.1	Monoids, Posets, and Groupoids	167
10.2	Actions	171
10.3	Reversible Graphs	176
10.4	Chaotic Graphs	180

10.5	Feedback and Control	186
10.6	To and from Idempotents	189
10.7	Additional Exercises	191
Appendixes		193
A	Logic as the Algebra of Parts	193
A.0	Why Study Logic?	193
A.1	Basic Operators and Their Rules of Inference	195
A.2	Fields, Nilpotents, Idempotents	212
B	The Axiom of Choice and Maximal Principles	220
C	Definitions, Symbols, and the Greek Alphabet	231
C.1	Definitions of Some Mathematical and Logical Concepts	231
C.2	Mathematical Notations and Logical Symbols	251
C.3	The Greek Alphabet	252
<i>Bibliography</i>		253
<i>Index</i>		257