

C O N T E N T S

1 ■	FUNDAMENTAL PRINCIPLES OF COUNTING	
1.1	The Rules of Sum and Product	2
1.2	Permutations	4
1.3	Combinations: No More Fuss About Order	11
1.4	Combinations with Repetition: Distributions	18
1.5	An Application in the Physical Sciences (Optional)	23
1.6	Summary and Historical Review	24
2 ■	ENUMERATION IN SET THEORY	
2.1	Fundamentals of Logic	30
2.2	Sets and Subsets	39
2.3	Set Operations and the Laws of Set Theory	45
2.4	Counting and Venn Diagrams	54
2.5	A Word on Probability	57
2.6	Summary and Historical Review	60
3 ■	RELATIONS AND FUNCTIONS	
3.1	Cartesian Products and Relations	66
3.2	Functions: Plain and One-to-One	71
3.3	Onto Functions: Stirling Numbers of the Second Kind	75
3.4	The Pigeonhole Principle	80
3.5	Special Functions	83
3.6	Summary and Historical Review	94
4 ■	LANGUAGES: FINITE STATE MACHINES	
4.1	Language: The Set Theory of Strings	102
4.2	Finite State Machines: A First Encounter	107
4.3	Finite State Machines: A Second Encounter	115
4.4	Summary and Historical Review	121

5	■	RELATIONS: THE SECOND TIME AROUND	
5.1		Relations Revisited: Properties of Relations	126
5.2		Computer Recognition: Zero-One Matrices and Directed Graphs	131
5.3		Partial Orders: Hasse Diagrams	141
5.4		Equivalence Relations and Partitions	148
5.5		Finite State Machines: The Minimization Process	151
5.6		Summary and Historical Review	158
6	■	THE SYSTEM OF INTEGERS	
6.1		The Well-Ordering Principle: Mathematical Induction	166
6.2		The Division Algorithm: Prime Numbers	174
6.3		The Greatest Common Divisor: The Euclidean Algorithm	177
6.4		The Fundamental Theorem of Arithmetic	182
6.5		Summary and Historical Review	184
7	■	THE PRINCIPLE OF INCLUSION AND EXCLUSION	
7.1		The Principle of Inclusion and Exclusion	190
7.2		Generalizations of the Principle	199
7.3		Derangements: Nothing Is in Its Right Place	202
7.4		Rook Polynomials	205
7.5		Arrangements with Forbidden Positions	208
7.6		Summary and Historical Review	213
8	■	RINGS AND MODULAR ARITHMETIC	
8.1		The Ring Structure: Definition and Examples	218
8.2		Ring Properties and Substructures	223
8.3		The Integers Modulo n	228
8.4		Ring Homomorphisms and Isomorphisms	233
8.5		Summary and Historical Review	238

- 9 ■ BOOLEAN ALGEBRA AND SWITCHING FUNCTIONS**
- 9.1 Switching Functions: Disjunctive and Conjunctive Normal Forms **244**
 - 9.2 Gating Networks: Minimal Sums of Products: Karnaugh Maps **251**
 - 9.3 Further Applications: Don't Care Conditions **260**
 - 9.4 The Structure of a Boolean Algebra (Optional) **266**
 - 9.5 Summary and Historical Review **273**
- 10 ■ GENERATING FUNCTIONS**
- 10.1 Introductory Examples **280**
 - 10.2 Definition and Examples: Computational Techniques **283**
 - 10.3 Partitions of Integers **289**
 - 10.4 The Exponential Generating Function **292**
 - 10.5 The Summation Operator **296**
 - 10.6 Summary and Historical Review **298**
- 11 ■ RECURRENCE RELATIONS**
- 11.1 The First-Order Linear Recurrence Relation **302**
 - 11.2 The Second-Order Linear Homogeneous Recurrence Relation with Constant Coefficients **308**
 - 11.3 The Nonhomogeneous Recurrence Relation **317**
 - 11.4 The Method of Generating Functions **322**
 - 11.5 A Special Kind of Nonlinear Recurrence Relation **326**
 - 11.6 Summary and Historical Review **330**
- 12 ■ GROUPS, CODING THEORY, AND POLYA'S METHOD OF ENUMERATION**
- 12.1 Definition, Examples, and Elementary Properties **336**
 - 12.2 Homomorphisms, Isomorphisms, and Cyclic Groups **341**
 - 12.3 Cosets and Lagrange's Theorem **345**
 - 12.4 Elements of Coding Theory **347**

12.5	The Hamming Metric	352	
12.6	The Parity-Check and Generator Matrices		354
12.7	Group Codes: Decoding with Coset Leaders		359
12.8	Hamming Matrices	363	
12.9	Counting and Equivalence: Burnside's Theorem		366
12.10	The Cycle Index	374	
12.11	The Pattern Inventory: Polya's Method of Enumeration	378	
12.12	Summary and Historical Review	384	

13 ■ FINITE FIELDS AND COMBINATORIAL DESIGNS

13.1	Polynomial Rings	390	
13.2	Irreducible Polynomials: Finite Fields		395
13.3	Latin Squares	402	
13.4	Finite Geometries and Affine Planes		407
13.5	Block Designs and Projective Planes		413
13.6	Summary and Historical Review	418	

14 ■ AN INTRODUCTION TO GRAPH THEORY

14.1	Definitions and Examples	424	
14.2	Subgraphs, Complements, and Graph Isomorphism	430	
14.3	Vertex Degree: Euler Paths and Cycles		439
14.4	Planar Graphs	445	
14.5	Hamilton Paths and Cycles	458	
14.6	Graph Coloring and Chromatic Polynomials		466
14.7	Summary and Historical Review	473	

15 ■ TREES

15.1	Definitions, Properties, and Examples		482
15.2	Rooted Trees	486	
15.3	Weighted Trees and Prefix Codes	503	
15.4	Biconnected Components and Articulation Points		508
15.5	Summary and Historical Review	514	

16 ■ OPTIMIZATION AND MATCHING

- 16.1 Minimal Spanning Trees: The Algorithms of Kruskal and Prim **518**
- 16.2 Transport Networks: The Max-Flow Min-Cut Theorem **523**
- 16.3 Matching Theory **535**
- 16.4 Summary and Historical Review **544**

■ ANSWERS A-1**■ INDEX I-1**