Contents

Preface v

Notation and Abbreviation vii

Contents xi

1. Permutations and Combinations 1

1.1. Two Basic Counting Principles 1

1.2. Permutations 6

1.3. Circular Permutations 12

1.4. Combinations 17

1.5. The Injection and Bijection Principles 27

1.6. Arrangements and Selections with Repetitions 32

1.7. Distribution Problems 40

Exercise 1 50

2. Binomial Coefficients and Multinomial Coefficients 69

2.1. Introduction 69

2.2. The Binomial Theorem 70

2.3. Combinatorial Identities 71

2.4. The Pascal's Triangle 76

2.5. Chu Shih-Chieh's Identity 78

2.6. Shortest Routes in a Rectangular Grid 85

2.7. Some Properties of Binomial Coefficients 93

2.8. Multinomial Coefficients and the Multinomial Theorem . . 96

Exercise 2 102

3. The Pigeonhole Principle and Ramsey Numbers 119

3.1. Introduction 119

3.2. The Pigeonhole Principle 119

3.3. More Examples 122

3.4. Ramsey Type Problems and Ramsey Numbers 129

3.5. Bounds for Ramsey Numbers 132

Exercise 3 137

4. The Principle of Inclusion and Exclusion 145
4.1. Introduction 145
4.2. The Principle 146
4.3. A Generalization 148
4.4. Integer Solutions and Shortest Routes 153
4.5. Surjective Mappings and Stirling Numbers of the Second Kind 158
4.6. Derangements and A Generalization 160
4.7. The Sieve of Eratosthenes and Euler φ-function 163
4.8. The 'Probléme des Ménages' 169
Exercise 4 173

5. Generating Functions 185
5.1. Ordinary Generating Functions 185
5.2. Some Modelling Problems 192
5.3. Partitions of Integers 196
5.4. Exponential Generating Functions 204
Exercise 5 211

6. Recurrence Relations 225
6.1. Introduction 225
6.2. Two Examples 228
6.3. Linear Homogeneous Recurrence Relations 234
6.4. General Linear Recurrence Relations 241
6.5. Two Applications 244
6.6. A System of Linear Recurrence Relations 251
6.7. The Method of Generating Functions 255
6.8. A Nonlinear Recurrence Relation and Catalan Numbers . . 259
6.9. Oscillating Permutations and an Exponential Generating Function 262
Exercise 6 270

Bibliography 287

Answers 289

Index 297