

# Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
1.1	Combinatorial Problems . . . . .	1
1.1.1	Links with linear programming . . . . .	3
1.2	Local and Global Optima . . . . .	4
1.3	Heuristics . . . . .	5
1.3.1	The case for heuristics . . . . .	7
1.3.2	Modern methods . . . . .	11
1.3.3	Evaluation of heuristics . . . . .	14
<b>2</b>	<b>Simulated Annealing</b>	<b>20</b>
2.1	Introduction . . . . .	20
2.2	The Basic Method . . . . .	24
2.2.1	Local optimization . . . . .	24
2.2.2	The annealing algorithm . . . . .	26
2.2.3	A brief overview of the theory . . . . .	27
2.2.4	Generic decisions . . . . .	29
2.2.5	Problem-specific decisions . . . . .	32
2.2.6	Examples . . . . .	34
2.2.7	Aids to fine-tuning . . . . .	40
2.3	Enhancements and Modifications . . . . .	42
2.3.1	Acceptance probability . . . . .	43
2.3.2	Cooling . . . . .	44
2.3.3	The neighbourhoods . . . . .	46
2.3.4	Sampling . . . . .	48
2.3.5	The cost function . . . . .	49
2.3.6	In combination with other methods . . . . .	51
2.3.7	Parallel implementations . . . . .	53
2.4	Applications . . . . .	55

2.4.1	Classical problems . . . . .	56
2.4.2	VLSI and computer design . . . . .	57
2.4.3	Sequencing and scheduling . . . . .	59
2.4.4	Other problems . . . . .	62
2.5	Conclusions . . . . .	63
<b>3</b>	<b>Tabu Search</b>	<b>70</b>
3.1	Introduction . . . . .	70
3.2	The Tabu Search Framework . . . . .	71
3.2.1	An illustrative example . . . . .	71
3.2.2	Notation and problem description . . . . .	82
3.2.3	Neighbourhood search . . . . .	83
3.2.4	Tabu search characteristics . . . . .	85
3.2.5	Tabu search memory . . . . .	88
3.2.6	Recency-based tabu memory functions . . . . .	94
3.2.7	Aspiration criteria . . . . .	98
3.2.8	Frequency-based memory . . . . .	104
3.2.9	Frequency-based memory in simple intensifica- tion and diversification processes . . . . .	109
3.3	Broader Aspects of Intensification and Diversification .	111
3.3.1	Diversification versus randomization . . . . .	112
3.3.2	Reinforcement by restriction . . . . .	114
3.3.3	Extrapolated relinking . . . . .	117
3.3.4	Solutions evaluated but not visited . . . . .	118
3.3.5	Interval-specific penalties and incentives . . . . .	119
3.3.6	Candidate list procedures . . . . .	119
3.3.7	Compound neighbourhoods . . . . .	121
3.3.8	Creating new attributes—vocabulary building and concept formation . . . . .	122
3.3.9	Strategic oscillation . . . . .	125
3.4	Tabu Search Applications . . . . .	127
3.5	Connections and conclusions . . . . .	136
3.5.1	Simulated annealing . . . . .	136
3.5.2	Genetic algorithms . . . . .	138
3.5.3	Neural networks . . . . .	141
<b>4</b>	<b>Genetic Algorithms</b>	<b>151</b>
4.1	Introduction . . . . .	151
4.2	Basic Concepts . . . . .	152

4.2.1	Intrinsic parallelism and the schema theorem . . . . .	154
4.2.2	Recent developments . . . . .	159
4.3	A Simple Example . . . . .	162
4.4	Extensions and Modifications . . . . .	164
4.4.1	Population-related factors . . . . .	165
4.4.2	Modified operators . . . . .	170
4.4.3	Chromosome coding and representation . . . . .	175
4.4.4	Hybridization . . . . .	179
4.4.5	Parallel implementations . . . . .	180
4.4.6	Computer software . . . . .	181
4.5	Applications . . . . .	181
4.5.1	Travelling salesman problem . . . . .	181
4.5.2	Sequencing and scheduling . . . . .	182
4.5.3	Graph colouring . . . . .	184
4.5.4	Steiner trees . . . . .	184
4.5.5	Knapsack problems . . . . .	185
4.5.6	Set covering problems . . . . .	185
4.5.7	Bin packing . . . . .	186
4.5.8	Neural networks . . . . .	186
4.5.9	Other problems . . . . .	187
4.6	Conclusions . . . . .	188
<b>5</b>	<b>Artificial Neural Networks</b>	<b>197</b>
5.1	Introduction . . . . .	197
5.2	Neural Networks . . . . .	198
5.2.1	Biological neural networks . . . . .	198
5.2.2	Artificial neural networks . . . . .	201
5.3	Combinatorial Optimization Problems . . . . .	204
5.4	The Graph Bisection Problem . . . . .	205
5.4.1	Neural mapping . . . . .	205
5.4.2	The mean field equations . . . . .	207
5.4.3	Mean field dynamics . . . . .	210
5.5	The Graph Partition Problem . . . . .	211
5.5.1	Neuron multiplexing—Ising representation . . . . .	211
5.5.2	K-state neurons—Potts representation . . . . .	213
5.5.3	Mean field Potts equations . . . . .	214
5.5.4	Mean field dynamics . . . . .	215
5.5.5	A generic algorithm . . . . .	220
5.5.6	Numerical results . . . . .	221

5.6	The Travelling Salesman Problem . . . . .	221
5.7	Scheduling Problems . . . . .	223
5.7.1	A synthetic example . . . . .	223
5.7.2	A realistic example . . . . .	226
5.8	Deformable Templates . . . . .	229
5.9	The Knapsack Problem . . . . .	233
5.10	Summary . . . . .	239
<b>6</b>	<b>Lagrangean Relaxation</b>	<b>243</b>
6.1	Introduction . . . . .	243
6.2	Overview . . . . .	245
6.2.1	Techniques . . . . .	245
6.2.2	Review of the literature . . . . .	246
6.3	Basic Methodology . . . . .	246
6.3.1	Introduction . . . . .	246
6.3.2	Lagrangean relaxation . . . . .	247
6.3.3	Set covering problem . . . . .	249
6.3.4	Example Lagrange multiplier values . . . . .	253
6.3.5	Advanced Lagrangean relaxation . . . . .	253
6.4	Lagrangean Heuristics and Problem Reduction . . . . .	260
6.4.1	Lagrangean heuristic . . . . .	260
6.4.2	Problem reduction . . . . .	261
6.4.3	Remarks . . . . .	266
6.5	Determination of Lagrange Multipliers . . . . .	266
6.5.1	Subgradient optimization . . . . .	267
6.5.2	Advanced subgradient optimization . . . . .	270
6.5.3	Multiplier adjustment . . . . .	273
6.6	Dual Ascent . . . . .	276
6.6.1	Basic concepts . . . . .	276
6.6.2	Example dual ascent algorithm . . . . .	278
6.6.3	Connections . . . . .	279
6.6.4	Comparisons . . . . .	280
6.6.5	Conclusions . . . . .	281
6.7	Tree Search . . . . .	282
6.7.1	Tree structure . . . . .	283
6.7.2	Branching node . . . . .	283
6.7.3	Backtracking . . . . .	283
6.7.4	Forward branching rule . . . . .	283
6.7.5	Lower bound computation . . . . .	285

- 6.7.6 Computer programs . . . . . 286
- 6.8 Applications . . . . . 290
  - 6.8.1 Journals . . . . . 290
  - 6.8.2 Summary . . . . . 297
- 6.9 Conclusions . . . . . 298
- 7 Evaluation of Heuristic Performance 304**
  - 7.1 Introduction . . . . . 304
  - 7.2 Analytical Methods . . . . . 305
    - 7.2.1 Worst-case and average performance analysis . 305
    - 7.2.2 Bounds . . . . . 307
  - 7.3 Empirical Testing . . . . . 308
  - 7.4 Statistical Inference . . . . . 311
  - 7.5 Conclusion . . . . . 313