

Contents

Introduction	1
1 Ramsey Theory and Topological Dynamics	5
1.1 Preliminaries	5
1.2 Van der Waerden's theorem	7
1.3 Gallai's theorem	11
1.4 A polynomial van der Waerden theorem	16
1.5 \mathbf{Z}_k and IP van der Waerden theorems	20
1.6 The Hales-Jewett coloring theorem	26
1.7 Recurrence for VIP-systems	29
1.8 The Bergelson-Leibman coloring theorem	33
2 Infinitary Ramsey Theory	40
2.1 Ramsey's theorem and Schur's theorem	40
2.2 Hindman's theorem	44
2.3 The Carlson-Simpson theorem	50
2.4 Carlson's theorem	57
2.5 Central sets	62
2.6 An infinitary set-polynomial theorem	68
3 Density Ramsey Theory	78
3.1 Measure theoretic preliminaries	78
3.2 Furstenberg correspondence	84
3.3 Kriz' example	89
3.4 Hilbert spaces	93
3.5 Sárközy's theorem	100
3.6 Polynomial recurrence along IP-sets	103

4 Three Ergodic Roth Theorems	111
4.1 Ergodicity and weak mixing	111
4.2 Roth's theorem	115
4.3 Measurable decomposition.....	119
4.4 Two dimensional Roth theorem.....	123
4.5 Double recurrence along IP sets	128
5 Multiple Recurrence	136
5.1 Furstenberg's structure theorem	136
5.2 Szemerédi's theorem.....	141
5.3 A polynomial Szemerédi theorem.....	145
References	153
List of Symbols	157
Index	158