

# CONTENTS

<b>Preface</b>	v
<b>1 Basic concepts</b>	1
1 The Notion of a Category. Duality. Subcategories. Examples.	1
2 Monomorphisms, epimorphisms, isomorphisms	3
3 Functors	7
4 Representable functors	13
5 Adjoint functors	17
6 The notion of equivalence between categories	27
<b>2 Sums and products</b>	33
1 Direct sums and products	33
2 Kernel and cokernel	43
3 Grothendieck topologies and the general notion of a sheaf	46
<b>3 Inductive and projective limits</b>	51
1 The general notion of a projective or inductive limit	51
2 Existence of inductive or projective limits	54
3 Commutation of functors with projective and inductive limits	56
4 Characterization of adjoint functors	60
5 Prorepresentable functors	67
<b>4 Structures on the objects of a category</b>	74
1 Algebraic operations on the objects of a category. Homomorphisms	74
2 The existence of kernels for homomorphisms	81
3 Equivalence relations.	83
4 The general notion of a structure on the objects of a category	84
<b>5 General theory of Abelian categories</b>	87
1 Additive categories	87
2 Kernel and cokernel	89
3 The canonical factorization of a morphism	91
4 Pre-Abelian categories	92

5	Abelian categories . . . . .	97
6	Exact functors . . . . .	98
7	The isomorphism theorems in Abelian categories . . . . .	101
8	The conditions AB3, AB4, AB5 . . . . .	111
9	Generators . . . . .	113
10	Full embedding of a small Abelian category into a Grothendieck category . . . . .	115
<b>6</b>	<b>Injective and projective objects in Abelian categories . . . . .</b>	<b>124</b>
1	The notion of an injective (projective) object and its general properties . . . . .	124
2	Essential extensions . . . . .	129
3	Properties of injective envelopes . . . . .	132
4	Projective objects . . . . .	134
5	Localization in rings . . . . .	135
6	Characterization of Grothendieck categories . . . . .	144
7	The theorem of Krull–Remak–Schmidt . . . . .	154
8	The structure of injective objects in locally Noetherian categories . . . . .	162
9	Applications to the decomposition theories . . . . .	165
<b>7</b>	<b>Elements of homological algebra . . . . .</b>	<b>170</b>
1	Complexes, homology, cohomology . . . . .	170
2	Resolutions . . . . .	174
3	Derived functors . . . . .	186
4	Other properties of derived functors . . . . .	189
5	Homology and cohomology functors . . . . .	191
6	Other properties of homology and cohomology functors . . . . .	197
7	Examples of homology and cohomology functors . . . . .	201
8	The homological dimension of Abelian categories . . . . .	204
9	Minimal projective resolutions . . . . .	207
10	Relative homological algebra . . . . .	211
	<b>Bibliography . . . . .</b>	<b>217</b>
	<b>Index . . . . .</b>	<b>223</b>