

# Contents

Preface . . . . .	v
Introduction . . . . .	xii
Notes for the Reader . . . . .	xxi
<b>Chapter 0: Preliminaries</b> . . . . .	1
0.1 Category Theory . . . . .	1
0.2 Sheaf Theory . . . . .	8
0.3 Grothendieck Topologies . . . . .	12
0.4 Giraud's Theorem . . . . .	15
Exercises 0 . . . . .	18
<b>Chapter 1: Elementary Toposes</b> . . . . .	23
1.1 Definition and Examples . . . . .	23
1.2 Equivalence Relations and Partial Maps . . . . .	27
1.3 The Category $\mathcal{E}^{\text{op}}$ . . . . .	31
1.4 Pullback Functors . . . . .	35
1.5 Image Factorizations . . . . .	40
Exercises 1 . . . . .	43
<b>Chapter 2: Internal Category Theory</b> . . . . .	47
2.1 Internal Categories and Diagrams . . . . .	47
2.2 Internal Limits and Colimits . . . . .	50
2.3 Diagrams in a Topos . . . . .	53
2.4 Internal Profunctors . . . . .	59
2.5 Filtered Categories . . . . .	65
Exercises 2 . . . . .	72
<b>Chapter 3: Topologies and Sheaves</b> . . . . .	76
3.1 Topologies . . . . .	76
3.2 Sheaves . . . . .	81
3.3 The Associated Sheaf Functor . . . . .	84
3.4 $\text{sh}_j(\mathcal{E})$ as a Category of Fractions . . . . .	90
3.5 Examples of Topologies . . . . .	93
Exercises 3 . . . . .	99

<b>Chapter 4: Geometric Morphisms . . . . .</b>	103
4.1 The Factorization Theorem . . . . .	103
4.2 The Glueing Construction . . . . .	107
4.3 Diaconescu's Theorem . . . . .	112
4.4 Bounded Morphisms . . . . .	119
Exercises 4 . . . . .	132
<b>Chapter 5: Logical Aspects of Topos Theory . . . . .</b>	136
5.1 Boolean Toposes . . . . .	136
5.2 The Axiom of Choice . . . . .	140
5.3 The Axiom (SG) . . . . .	145
5.4 The Mitchell–Bénabou Language . . . . .	152
Exercises 5 . . . . .	161
<b>Chapter 6: Natural Number Objects . . . . .</b>	165
6.1 Definition and Basic Properties . . . . .	165
6.2 Finite Cardinals . . . . .	173
6.3 The Object Classifier . . . . .	180
6.4 Algebraic Theories . . . . .	190
6.5 Geometric Theories . . . . .	198
6.6 Real Number Objects . . . . .	210
Exercises 6 . . . . .	220
<b>Chapter 7: Theorems of Deligne and Barr . . . . .</b>	224
7.1 Points . . . . .	224
7.2 Spatial Toposes . . . . .	229
7.3 Coherent Toposes . . . . .	232
7.4 Deligne's Theorem . . . . .	240
7.5 Barr's Theorem . . . . .	249
Exercises 7 . . . . .	254
<b>Chapter 8: Cohomology . . . . .</b>	259
8.1 Basic Definitions . . . . .	259
8.2 Čech Cohomology . . . . .	266
8.3 Torsors . . . . .	272
8.4 Profinite Fundamental Groups . . . . .	283
Exercises 8 . . . . .	290
<b>Chapter 9: Topos Theory and Set Theory . . . . .</b>	296
9.1 Kuratowski-Finiteness . . . . .	296
9.2 Transitive Objects . . . . .	303
9.3 The Equiconsistency Theorem . . . . .	312
9.4 The Filterpower Construction . . . . .	319
9.5 Independence of the Continuum Hypothesis . . . . .	323
Exercises 9 . . . . .	330

<b>Appendix: Locally Internal Categories . . . . .</b>	<b>334</b>
Bibliography . . . . .	347
Index of Definitions . . . . .	357
Index of Notation . . . . .	361
Index of Names . . . . .	366