

INTRODUCTIONP A R T ISPIN $\frac{1}{2}$ LATTICE SYSTEMS WITHOUT CONSTRAINTS

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SPIN $\frac{1}{2}$ LATTICE SYSTEMS WITH CONSTRAINTS

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