

Contents

1	Introduction	11
2	Preliminaries	15
2.1	Fuzzy Sets	15
2.1.1	Basic Notions	15
2.1.2	Ordering-Based Convexity	18
2.2	Logical Operations	22
2.2.1	Triangular Norms and Conorms	22
2.2.2	Negations	27
2.2.3	Implications	29
2.2.4	A Short Remark on Fuzzy Logic	34
2.3	The Extension Principle	35
2.4	Binary Fuzzy Relations	37
2.4.1	Basic Notions and Properties	37
2.4.2	Congruence and Hulls	39
2.4.3	Fuzzy Equivalence Relations	43
2.4.4	Fuzzy Orderings	47
3	Overcoming the “Crispness” of Fuzzy Orderings	51
3.1	A Critical View on the Existing Definitions	51
3.1.1	Implications as Orderings?	51
3.1.2	Inclusion Relations	53
3.1.3	The Fuzzification Property	54
3.1.4	Reflexivity versus Antisymmetry	56
3.2	Preserving the Classical Axioms by Adding Similarity	58
3.2.1	The Interpretation of Induced Similarities	58
4	Constructions and Representations	63
4.1	Applying Connectives to Fuzzy Orderings	63
4.1.1	Intersections and Unions	63
4.1.2	Compositions	65

4.1.3	Cartesian Products	67
4.2	Inverses and Duals	68
4.3	Factorization	73
4.4	The Fuzzification Property Revisited	75
4.4.1	Extracting Crisp From Fuzzy Orderings	75
4.4.2	Direct Fuzzifications of Crisp Orderings	81
5	From Hulls to Hedges	87
5.1	Motivation	87
5.2	Hulls with Respect to Direct Fuzzifications	89
5.3	Convex Hulls and their Characterization	92
5.4	The Role of the Extension Principle	95
5.5	More about Ordering-Based Hedges	97
6	Orderings of Fuzzy Sets	99
6.1	Motivation	99
6.2	A Novel Approach based on Fuzzy Orderings	100
6.2.1	Basic Properties	102
6.2.2	Connections to the Extension Principle	104
6.2.3	Weaknesses	106
6.3	Generalizations and Extensions	108
6.3.1	Fuzzification	108
6.3.2	Compensating different Heights	113
6.3.3	A Clue to Hybridization	118
6.4	The Monotonicity of Extended Monotonic Mappings	120
6.5	Classification according to Wang and Kerre	127
7	Conclusion and Outlook	133
Symbol Reference		135
Bibliography		139
Index		145