

Contents

1. Problems, Illustrations, History	1
1.1. The Four Color Problem	1
1.2. Map Color Theorem	2
1.3. The Thread Problem	5
1.4. Unilateral Surfaces	7
2. Graph Theory	10
2.1. Chromatic Number	10
2.2. Rotations of Graphs	16
2.3. Orientable Cases 7 and 10	25
3. Classification of Surfaces	34
3.1. The Concept of Topology	34
3.2. Polyhedra	35
3.3. Elementary Operations	40
3.4. Normal Form for Orientable Surfaces	42
3.5. Normal Form for Non-Orientable Surfaces	46
3.6. Standard Models	47
3.7. Partial Polyhedra	51
4. Graphs on Surfaces	54
4.1. Embedding Theorem	54
4.2. Dual Polyhedra	60
4.3. Heawood's Inequality	63
4.4. Genus of Graphs	65
4.5. Non-Orientable Genus of Graphs	66
4.6. Klein's Bottle	69
5. Combinatorics of Embeddings	74
5.1. Triangular Embeddings	74
5.2. Orientable Special Cases	79
5.3. Outline for General Cases	85

6. Orientable Cases 1, 4, and 9	89
6.1. Orientable Case 4	89
6.2. Arithmetic Combs	92
6.3. Orientable Case 1	93
6.4. Coil Diagrams	96
6.5. Orientable Case 9	99
7. Orientable Cases 11, 2, and 8	102
7.1. Example for $n=35$	102
7.2. Orientable Case 11	105
7.3. The Additional Adjacency Problem	110
7.4. Orientable Case 2	114
7.5. Additional Adjacency Problem	116
7.6. Orientable Case 8	120
8. Non-Orientable Cases (Index 1)	130
8.1. Method of Doubling	130
8.2. Non-Orientable Cases 0, 3, 7	134
8.3. Cascades	137
8.4. Orientable Application	144
9. Solutions of Index 2 and 3	146
9.1. Examples and Method	146
9.2. Orientable Cases 3 and 5	151
9.3. Orientable Case 6	154
9.4. Non-Orientable Case 9	157
10. Construction by Induction	159
10.1. An Index 3 Induction	159
10.2. An Index 2 Induction	164
10.3. Non-Orientable Cases 1, 2, 6, and 10	167
11. Orientable Case 0	169
11.1. Currents from Non-Abelian Groups	169
11.2. Examples	170
11.3. General Solution	174
12. Related Problems	178
12.1. Questions about Rotations	178
12.2. Questions about Embeddings	179
References	184
Index	189