List of Contents

Preface vii

Chapter I. Probability Theoretic Preliminaries 1

1. Notation and basic facts 1
2. Some basic distributions 5
3. Normal approximation 9
4 Sieve formulae 14
5. Convergence in distribution 22
   Exercises 27

Chapter II. Models of Random Graphs 31

1. The basic models 31
2. Properties of almost all graphs 40
3. Large subsets of vertices 43
4. Random regular graphs 47
   Exercises 53

Chapter III. The Degree Sequence 56

1. The distribution of an element of the degree sequence 56
2. Almost determined degrees 62
3. The shape of the degree sequence 65
4. Jumps and repeated values 68
5. Fast algorithms for the graph isomorphism problem 69
   Exercises 71

Chapter IV. Small Subgraphs 74

1. Strictly balanced graphs 75
2. Arbitrary subgraphs 81
3. Poisson approximation 86
   Exercises 88

Chapter V. The Evolution of Random Graphs—Sparse Components . . . 90

1. Trees of given sizes as components 91
2. The number of vertices on tree components 96
3. The largest tree components 103
4. Components containing cycles 111

Exercises 121

Chapter VI. The Evolution of Random Graphs—The Giant Component . . 123

1. A gap in the sequence of components 123
2. The emergence of the giant component 131
3. Small components after time $n/2$ 136
4. An application 141

Exercises 145

Chapter VII. Connectivity and Matchings. 146

1. The connectedness of random graphs 147
2. The k-connectedness of random graphs 152
3. Matchings in bipartite graphs 155
4. Matchings in random graphs 159
5. Reliable networks 168
6. Regular graphs 174

Exercises 177

Chapter VIII. Long Paths and Cycles 180

1. Long paths in $G_{c/n}$—first approach 181
2. Hamilton cycles—first approach 185
3. Hamilton cycles—second approach 190
4. Long paths in $G_{c/n}$—second approach 196
5. Hamilton cycles in regular graphs 198

Exercises 202

Chapter IX. The Automorphism Group 203

1. The number of unlabelled graphs 203
2. The asymptotic number of unlabelled regular graphs. 216
3. Distinguishing vertices by their distance sequences 218
4. Asymmetric graphs 219
5. Graphs with a given automorphism group. 222

Exercises 223

Chapter X. The Diameter 225

1. Large graphs of small diameter 225

2. The diameter of G_p 228
3. The diameter of random regular graphs 238
4. Graph processes 241
5. Related results 245
Exercises 249

Chapter XI. Cliques, Independent Sets and Colouring 251

1. Cliques in G_p 251
2. Poisson approximation 259
3. Colouring random graphs. 262
4. Sparse graphs 267
Exercises 277

Chapter XII. Ramsey Theory 280

1. Bounds on $R(s)$. 281
2. Off-diagonal Ramsey numbers 284
3. Triangle-free graphs 292
4. Dense subgraphs 299
5. The size-Ramsey number of a path 300
Exercises 304

Chapter XIII. Explicit Constructions 307

1. Character sums. 307
2. The Paley graph P_q 315
3. Dense graphs 323
4. Sparse graphs 330
Exercises 334

Chapter XIV. Sequences, Matrices and Permutations 336

1. Random subgraphs of the cube 337
2. Random matrices 347
3. Balancing families of sets 352
4. Random elements of finite groups 360
5. Random mappings 364
Exercises 373

Chapter XV. Sorting Algorithms 377

1. Finding most comparisons in one round 378
2. Sorting in two rounds 383
3. Sorting with width $n/2$ 387
4. Bin packing 393
Exercises 398

Chapter XVI. Random Graphs of Small Order 399

1. Connectivity 399
2. Independent sets 400
3. Colouring. 404
4. Regular graphs 407

References 410

Subject Index 442

Some Notation Used in the Text 446