

Contents

Preface

vii

I Basic Results	1
1.0 Introduction	1
1.1 Basic Concepts	2
1.2 Subgraphs	8
1.3 Degrees of Vertices	10
1.4 Paths and Connectedness	13
1.5 Automorphism of a Simple Graph	18
1.6 Line Graphs	20
1.7 Operations on Graphs	25
1.8 An Application to Chemistry	29
1.9 Miscellaneous Exercises	31
Notes	32
II Directed Graphs	33
2.0 Introduction	33
2.1 Basic Concepts	33
2.2 Tournaments	35
2.3 k -Partite Tournaments	38
Notes	43
III Connectivity	44
3.0 Introduction	44
3.1 Vertex Cuts and Edge Cuts	44

3.2	Connectivity and Edge-Connectivity	48
3.3	Blocks	54
3.4	Cyclical Edge-Connectivity of a Graph	56
3.5	Menger's Theorem	57
3.6	Exercises	65
	Notes	66
IV	Trees	67
4.0	Introduction	67
4.1	Definition, Characterization, and Simple Properties	67
4.2	Centers and Centroids	72
4.3	Counting the Number of Spanning Trees	75
4.4	Cayley's Formula	79
4.5	Helly Property	80
4.6	Exercises	81
	Notes	82
V	Independent Sets and Matchings	83
5.0	Introduction	83
5.1	Vertex Independent Sets and Vertex Coverings	83
5.2	Edge-Independent Sets	85
5.3	Matchings and Factors	87
5.4	Matchings in Bipartite Graphs	90
5.5*	Perfect Matchings and the Tutte Matrix	99
	Notes	101
VI	Eulerian and Hamiltonian Graphs	102
6.0	Introduction	102
6.1	Eulerian Graphs	102
6.2	Hamiltonian Graphs	107
6.3*	Pancyclic Graphs	115
6.4	Hamilton Cycles in Line Graphs	118
6.5	2-Factorable Graphs	124
6.6	Exercises	126
	Notes	127
VII	Graph Colorings	128
7.0	Introduction	128
7.1	Vertex Colorings	128
7.2	Critical Graphs	132
7.3	Triangle-Free Graphs	136
7.4	Edge Colorings of Graphs	138
7.5	Snarks	144
7.6	Kirkman's Schoolgirls Problem	145
7.7	Chromatic Polynomials	147
	Notes	150

VIII	Planarity	152
	8.0 Introduction	152
	8.1 Planar and Nonplanar Graphs	152
	8.2 Euler Formula and Its Consequences	157
	8.3 K_5 and $K_{3,3}$ are Nonplanar Graphs	162
	8.4 Dual of a Plane Graph	164
	8.5 The Four-Color Theorem and the Heawood Five-Color Theorem	167
	8.6 Kuratowski's Theorem	170
	8.7 Hamiltonian Plane Graphs	178
	8.8 Tait Coloring	180
	Notes	184
IX	Triangulated Graphs	185
	9.0 Introduction	185
	9.1 Perfect Graphs	185
	9.2 Triangulated Graphs	187
	9.3 Interval Graphs	190
	9.4 Bipartite Graph $B(G)$ of a Graph G	192
	9.5 Circular Arc Graphs	193
	9.6 Exercises	193
	9.7 Phasing of Traffic Lights at a Road Junction	195
	Notes	198
X	Applications	199
	10.0 Introduction	199
	10.1 The Connector Problem	199
	10.2 Kruskal's Algorithm	200
	10.3 Prim's Algorithm	202
	10.4 Shortest-Path Problems	203
	10.5 Timetable Problem	205
	10.6 Application to Social Psychology	207
	10.7 Exercises	210
	Notes	210
	List of Symbols	213
	References	217
	Index	223