CONTENTS

FREFACE	Х
CHAPTER I. PREFERENCE ORDERINGS AND UTILITY THEORY	1
1. Relational systems	1
2. Preference relations	2
3. Some remarks on utility theory	8
3.1. On the formal notion of utility theory	g
4. Linear inequalities	11
4.1. Theorems of alternatives	12
4.2. An application	19
CHAPTER II. ORDINAL UTILITY	21
1. Some classical representation theorems	21
2. Lexicographic utility	26
3. Utility theories with respect to η_{α} -sets	28
4. Ultraproducts and ultrapowers	32
4.1. Some definitions and properties	32
4.2. Introducing non-standard models of the reals	35
4.3. Ultrapowers of the reals over a countable index set	36
4.4. Non-standard models of the rational numbers which are	
also η_1 -sets	38
5. Approximating an R^* -valued utility function by a real valued	
function	42
6. Non-standard utility functions always exist	43
7. Utility functions for partial orderings	45
7.1. Utility functions in the wider sense	45
7.2. Utility functions in the narrower sense	49
CHAPTER III. ON NUMERICAL RELATIONAL SYSTEMS	53
1. First-order languages	53
2. Some preliminary considerations	60

VIII CONTENTS

3.	. Universal and homogeneous relational systems	61
	Saturated relational systems	64
	4.1. Some fundamental results	64
	4.2. Special relational systems	67
	4.3. Some special results for complete theories	69
	4.4. Ultraproducts and saturated relational systems	71
C	HAPTER IV. UTILITY THEORIES FOR MORE STRUCTURED	
	EMPIRICAL DATA	76
1.	Some remarks	76
2.	The empirical status of axioms	77
	Utility theories which are axiomatizable in an ordinary first	
	order language	79
	3.1. Utility theories which are universally axiomatizable	79
	3.2. On suitable numerical relational systems for utility	
	theories which are axiomatizable by finitely many	
	universal sentences	82
4.	Extensive utility	85
	4.1. Hölder's theorem	86
	4.2. On the existence of real-valued utility functions for	
	ordered semigroups	88
	4.3. Non-Archimedean extensive utility	90
	4.3.1. On the empirical status of the axioms of ordered	
	Abelean groups and the existence of utility functions	91
	4.3.2. The divisible ordered Abelean groups	92
	4.3.3. An application of Robinson's model completeness	
_	test	95
	Conjoint measurement of utilities	99
0.	On certain mean systems	101
	6.1. Archimedean mean systems	101
	6.2. Non-Archimedean mean systems	103
CI	HAPTER V. ON UTILITY SPACES, THE THEORY OF	
	GAMES AND THE REALIZATION OF	
	COMPARATIVE PROBABILITY RELATIONS	109
1.	A generalization of the von Neumann/Morgenstern utility	
	theory	109

CONTENTS	IX
 Non-standard utilities in game theory Some aspects of the realization of comparative probability 	111
relations	114
3.1. Boolean algebras and fields of sets	114
3.2. On the realization of some comparative probability	
relations	116
APPENDIX I. ORDINAL AND CARDINAL NUMBERS	124
APPENDIX II. SOME BASIC FACTS ABOUT FILTERS AND ULTRAFILTERS	126
BIBLIOGRAPHY	128
INDEX OF NAMES	134
INDEX OF SUBJECTS	136