## Contents

| FOREWORD<br>PREFACE<br>LIST OF PRINCIPAL SYMBOLS              | ix<br>xi<br>xv |  |  |
|---|----------------|--|--|
| CHAPTER I FUNDAMENTAL NOTIONS                                 |                |  |  |
| 1. Introduction   | 1              |  |  |
| 2. Review of the notion of membership                         |                |  |  |
| 3. The concept of a fuzzy subset                              |                |  |  |
| 4. Dominance relations  | 4              |  |  |
| 5. Simple operations on fuzzy subsets                         | 8              |  |  |
| 6. Set of fuzzy subsets for E and M finite                    | 30             |  |  |
| 7. Properties of the set of fuzzy subsets                     | 33             |  |  |
| 8. Product and algebraic sum of two fuzzy subsets             | 34             |  |  |
| 9. Exercises  | 38             |  |  |
| CHAPTER II FUZZY GRAPHS AND FUZZY REL                         | ATIONS         |  |  |
| 10. Introduction  | 41             |  |  |
| 11. Fuzzy graphs  | 41             |  |  |
| 12. Fuzzy relations   | 46             |  |  |
| 13. Composition of fuzzy relations                            | 60             |  |  |
| 14. Fuzzy subsets induced by a mapping                        | 69             |  |  |
| 15. Conditioned fuzzy subsets                                 | 71             |  |  |
| 16. Properties of fuzzy binary relations                      |                |  |  |
| 17. Transitive closure of a fuzzy binary relation             | 87             |  |  |
| 18. Paths in a finite fuzzy graph                             | 94             |  |  |
| 19. Fuzzy preorder relations                                  | 98             |  |  |
| 20. Similitude relations                                      | 101            |  |  |
| 21. Similitude subrelations in a fuzzy preorder               | 104            |  |  |
| 22. Antisymmetry  | 105            |  |  |
| 23. Fuzzy order relations                                     | 110            |  |  |
| 24. Antisymmetric relations without loops. Ordinal relations. |                |  |  |
| Ordinal functions in a fuzzy order relation                   | 118            |  |  |
| 25. Dissimilitude relations                                   | 127            |  |  |
| 6. Resemblance relations                                      |                |  |  |
| 7. Various properties of similitude and resemblance           |                |  |  |
| 28. Various properties of fuzzy perfect order relations       | 160            |  |  |
| 29. Ordinary membership functions                             | 167            |  |  |
| 30. Exercises   | 180            |  |  |

ri CONTENTS

## CHAPTER III FUZZY LOGIC

| 31         | Introductio  | n  | 191 |
|------------|--------------|--|-----|
|            |              | ic function of a fuzzy subset. Fuzzy variables   | 192 |
|            | Polynomial   |  | 201 |
| 34         | Analysis of  | a function of fuzzy variables. Method of Marinos   | 208 |
| 35         | Logical stru | cture of a function of fuzzy variables   | 214 |
|            |              | n of intervals   | 218 |
|            |              | f a function of fuzzy variables  | 225 |
|            |              | f fuzzy elements   | 235 |
| 30.        | Fuzzy prop   | ositions and their functional representations  | 243 |
| 40         | The theory   | of fuzzy subsets and the theory of probability   | 250 |
| 41         | The theory   | of fuzzy subsets and the theory of functions of structure  | 253 |
|            | Exercises    | or rainly shows and the  | 264 |
| 72.        | LACTOISCS    |  |     |
|            | (            | CHAPTER IV THE LAWS OF FUZZY COMPOSITION   |     |
| 43.        | Introduction | on   | 267 |
|            |              | the notion of a law of composition   | 267 |
| 45.        | Laws of fuz  | zzy internal composition. Fuzzy groupoids  | 269 |
| 46.        | Principal p  | roperties of fuzzy groupoids   | 273 |
|            | Fuzzy mon    |  | 278 |
|            | •            | rnal composition   | 284 |
|            |              | on fuzzy numbers   | 290 |
|            | Exercises    |  | 295 |
|            | CHAPTER      | V GENERALIZATION OF THE NOTION OF FUZZY SUBSET   |     |
|            |              |  | 297 |
|            | Introduction |  | 298 |
| 52.        | . Operations | on ordinary sets tal properties of the set of mappings from one set to another                   | 301 |
| 53.        | . Fundamen   | several fundamental structures   | 304 |
|            |              |  | 315 |
|            |              | tion of the notion of fuzzy subsets  | 334 |
| 56.        | . Operations | s on fuzzy subsets when L is a lattice various notions with a view toward explaining the concept |     |
| 57         |              |  | 337 |
| <b>5</b> 0 | of category  |  | 358 |
|            |              | pt of a category   | 370 |
|            | . Fuzzy C-M  | iorpnisms  | 378 |
| 60         | . Exercises  |  |     |
| Ar         | pendix A     | General proof procedure for operations concerning  |     |
| <b>r</b>   | •            | maxima and minima  | 383 |
| Αı         | opendix B    | Decomposition into maximal similitude subrelations   | 387 |
| 1          | r            | •  |     |

|              | CONTENTS | vii |
|--------------|----------|-----|
|              |          | 401 |
| Conclusion   |          | 401 |
| Bibliography |          | 403 |
| References   |          | 411 |

413

Index