

Contents

COMMENTS FOR THE INSTRUCTOR		xi
	To the Student	xiv
1	NUMBERS AND PROGRAMS	1
1.1	The Basics of ISETL	1
	Activities	1
	Discussion	1
	Beginning with ISETL	1
	Some Syntax	3
	Familiar Sets of Numbers	3
	Decimal Representation	5
	Binary Representation	5
	Sequences	6
	Exercises	6
1.2	Divisibility	9
	Activities	9
	Discussion	12
	ISETL <i>funcs</i> —Functions	12
	ISETL <i>smaps</i> —Functions	13
	Sources of Functions	14
	Recursive Functions	15
	Modular Arithmetic	15
	Prime Numbers	16
	Common Divisors	18
	Common Multiples	20
	Exercises	21
	Overview of Chapter 1	24
2	PROPOSITIONAL CALCULUS	27
2.1	Boolean Expressions	27
	Activities	27
	Discussion	29
	Constants and Variables	29
	Basic Operations	30
	Functions Using Boolean Values	32
	Exercises	33
2.2	Implication and Proof	34
	Activities	34

Discussion	36
Conditional Statements	36
Variations of Conditional Statements	37
Direct Proof	38
Indirect Proof	38
Proof by Contradiction	39
Exercises	40
Overview of Chapter 2	41
3 SETS AND TUPLES	43
3.1 Defining Sets and Tuples	43
Activities	43
Discussion	46
Sets and their Elements	46
Tuples and their Elements	48
Forming Sets and Tuples	48
Sequences	50
Recursive Sequences	50
Exercises	51
3.2 Operations on Sets	53
Activities	53
Discussion	55
Cardinality	55
Subsets	55
Basic Combinations of Sets	57
De Morgan's Laws	58
Cartesian Products	59
Inclusion-Exclusion	59
Exercises	60
3.3 Counting Methods	62
Activities	62
Discussion	64
The Multiplication Principle	64
Permutations	65
Combinations	66
The Pigeonhole Principle	67
Exercises	68
Overview of Chapter 3	70
4 PREDICATE CALCULUS	73
4.1 Quantified Expressions	73
Activities	73
Discussion	76
Existential and Universal Quantifiers	76

	Quantifying over Proposition Valued Functions—	
	Existential	76
	Quantifying over Proposition Valued Functions—	
	Universal	77
	Negations	78
	Reasoning about Quantified Expressions	78
	Exercises	79
4.2	Multi-Level Quantification	84
	Activities	84
	Discussion	87
	Quantified Statements that Depend on a Variable	87
	Two-Level Quantification	89
	Negating Two-Level Quantifications	90
	Reasoning about Two-Level Quantifications	91
	Three-Level Quantification	92
	Exercises	92
	Overview of Chapter 4	96
5	RELATIONS AND GRAPHS	97
5.1	Relations and their Graphs	97
	Activities	97
	Discussion	99
	Relations	99
	Representing a Relation	100
	Properties of Relations	101
	More about Graphs	102
	Exercises	103
5.2	Equivalence Relations and Graph Theory	106
	Activities	106
	Discussion	107
	Equivalence Relations	107
	Types of Graphs	109
	Subgraphs	109
	Planarity	111
	Exercises	111
	Overview of Chapter 5	114
6	FUNCTIONS	117
6.1	Representing Functions	117
	Activities	117
	Discussion	120
	Constructing Functions	120
	Functions as Expressions	122
	Functions as Sequences	122
	Functions as Tables	123

	Functions as Graphs	124
	The Process of a Function	125
	Two Definitions	126
	Exercises	126
6.2	Properties of Functions	129
	Activities	129
	Discussion	132
	Basic Properties	132
	One-to-One Functions	133
	Combinations of Functions	135
	Inverse Functions	137
	Rate of Growth for Functions	138
	Exercises	140
	Overview of Chapter 6	143
7	MATHEMATICAL INDUCTION	145
7.1	Understanding the Method	145
	Activities	145
	Discussion	148
	Proposition-Valued Functions	148
	Eventually Constant Proposition-Valued Functions	148
	Implication-Valued Functions	149
	Modus Ponens	150
	Coordinating the Steps	151
	Exercises	151
7.2	Using Mathematical Induction	153
	Activities	153
	Discussion	154
	Making Induction Proofs	154
	The Induction Principle	156
	Complete Induction	156
	The Binomial theorem	157
	Exercises	158
	Overview of Chapter 7	160
8	PARTIAL ORDERS	163
	Activities	163
	Discussion	164
	Order on a Set	164
	Diagrams of Posets	165
	Topological Sorting	166
	Sperner's Theorem	167
	Exercises	168
	Overview of Chapter 8	170

9 INFINITE SETS	173
Discussion	173
Sets of Equal Cardinality	173
Infinite Sets	174
Countable Sets	174
Uncountable Sets	178
Ordering of Infinite Sets	179
Exercises	180
APPENDIX 1: GETTING STARTED WITH ISETL	182
A. Working in the Execution Window	182
B. Working with Files	184
C. Using Directives	185
D. Graphing in ISETL	186
APPENDIX 2: SOME SPECIAL CODE	188
INDEX	191
INDEX OF FREQUENTLY USED SETS AND FUNCTIONS	194