

Table of Contents

I. GROUPS	1
1. Sets and Mappings	1
2. Products and Operations	3
3. Relations	4
4. Properties of the Integers	5
5. Groups	7
6. Elementary Properties of Groups	7
7. Subgroups	9
8. Cyclic and Finite Groups	10
9. Cosets and Normal Subgroups	11
10. Homomorphism	13
11. Quotient Groups	14
12. The Fundamental Theorem	16
13. Direct Products	17
14. Finite Cyclic Groups	19
15. Finite Abelian Groups	20
16. Permutation Groups	22
17. Computation of Transforms	24
18. The Regular Representation of a Group	25
19. Composition Series	26
20. References	28
I. RINGS AND FIELDS	29
1. Rings	29
2. Ring Homomorphism	30
3. The Characteristic of a Ring	31
4. Ideals	33
5. Difference Rings	35
6. Polynomials over a Ring	37
7. Construction of $\mathfrak{A}[x]$	37
8. The Division Algorithm	38
9. Polynomials in n Independent Indeterminates	40
10. Integral Domains and Fields	41
11. Divisibility in a Commutative Ring	43
12. Prime and Maximal Ideals	44
13. The Ring of Integers	45
14. Factorization Theory in $\mathfrak{F}[x]$	48
15. Ideal Theory in $\mathfrak{F}[x]$	50
16. References	51

III. VECTOR SPACES AND MATRICES	52
1. Vector Spaces	52
2. Sums of Subspaces	53
3. Vector Spaces over a Field	55
4. Linear Mappings	56
5. The Space of n -tuples	57
6. Matrices	58
7. The Vector Space $\mathfrak{M}_{m, n}$	61
8. The Matrix of a Linear Mapping	62
9. The Sign of a Permutation	64
10. Determinants	66
11. Non-singular Matrices	68
12. The Effect of a Change of Basis	68
13. Elementary Transformations over \mathfrak{J}	70
14. Equivalence in \mathfrak{F} and Determinantal Rank	72
15. Linear Systems	75
16. Equivalence in $\mathfrak{F}[x]$	78
17. The Ring \mathfrak{M}_n over $\mathfrak{F}[x]$	82
18. Similarity of Square Matrices	83
19. Characteristic and Minimum Functions	83
20. The Companion Matrix of a Polynomial	85
21. Nilpotent Matrices	88
22. The Jordan Canonical Form	88
23. The Characteristic Roots of a Polynomial	94
24. References	95
IV. THEORY OF ALGEBRAIC EXTENSIONS	96
1. Simple Extensions	96
2. Algebraic Extensions of Finite Degree	97
3. Subfields	97
4. Splitting Fields	98
5. Composites	100
6. Binomial and Trinomial Equations	101
7. Separable Polynomials and Fields	102
8. The Artin Lemmas	105
9. Normal Fields	106
10. Characterizations of Normality	108
11. The Galois Group of an Equation	111
12. The Fundamental Theorems of the Galois Theory	112
13. Simple Extensions	114
14. The Characteristic Function of a Field	118
15. The Normal Basis Theorem	119
16. The Trace Theorem	121
17. References	122

V. FINITE FIELDS	123
1. Number of Elements	123
2. Existence and Uniqueness	124
3. The Cyclic Group \mathfrak{F}^*	124
4. Primitive Roots Modulo m	125
5. The Galois Group of a Finite Field	127
6. Dedekind's Formula	128
7. Polynomials Belonging to an Exponent	130
8. Polynomials Belonging to a Prime	132
9. A Construction of Irreducible Polynomials	133
10. The Exceptional Case	135
11. Irreducible Polynomials of Even Degree	137
12. Applications	138
13. Polynomials of Degree p over \mathfrak{F}_q , $q = p^n$	140
14. Dickson's Theorem	141
15. Generation of Irreducible Polynomials	142
16. The Cubing Transformation	143
17. Determination of Primitive Irreducible Polynomials	145
18. Miscellaneous Results	149
19. Historical Notes	155
 APPENDIXES	
I. TABLE OF LEAST PRIMITIVE ROOTS	157
II. EXTRACT FROM JACOBI'S CANON	158
III. BUSSEY'S TABLES	159
IV. IRREDUCIBLE POLYNOMIALS OVER \mathfrak{F}_2	161
INDEX	163