

# CONTENTS

<b>Prerequisite Set Theory</b>	<b>1</b>
<b>Chapter 1 Groups</b>	<b>3</b>
1 The Definition of a Group,	3
2 Subgroups,	8
3 Cosets, Lagrange's Theorem, and Normal Subgroups,	11
4 Homomorphisms,	15
5 Direct and Free Products,	21
6 Free Groups and Presentations,	25
7 Exact Sequences,	27
<b>Chapter 2 Abelian Groups</b>	<b>38</b>
1 Special Features of Commutative Groups,	38
2 Direct Sums and Products of Abelian Groups; The Structure of Finitely Generated Abelian Groups,	42
3 Projective and Injective Abelian Groups,	46
4 Exact Sequences of Abelian Groups,	52
5 The Tensor Product of Abelian Groups,	54
<b>Chapter 3 Categories and Functors</b>	<b>64</b>
1 Categories,	65
2 Functors,	68
3 Natural Transformations,	72
4 Duality Principle,	74
5 Products and Coproducts,	80
6 Pullbacks and Pushouts,	89
7 Adjoint Functors,	98
8 Abelian Categories,	106

<b>Chapter 4 Modules</b>	<b>125</b>
1 Rings, 125	
2 Modules, 131	
3 The Functor $\text{Hom}$ , 137	
4 The Functor $\otimes$ , 140	
5 Projective Modules, 144	
6 Injective Modules, 151	
<b>Chapter 5 Integral Domains</b>	<b>159</b>
1 Principal Ideal Domains, 159	
2 Unique Factorization Domains, 163	
3 Noetherian Rings and Modules, 170	
4 Modules over Principal Ideal Domains, 173	
<b>Chapter 6 Semisimple Rings</b>	<b>181</b>
1 The Morita Theorem, 181	
2 Semisimple Rings, 184	
<b>Chapter 7 The Functors <math>\text{Ext}</math> and <math>\text{Tor}</math></b>	<b>191</b>
1 Chain Complexes, Chain Maps, and Homology, 191	
2 Chain Homotopy and the Fundamental Lemmas, 193	
3 The Functor $\text{Ext}$ , 201	
4 Properties of $\text{Ext}$ , 206	
5 The Functor $\text{Tor}$ , 214	
6 Pullbacks and Pushouts in ${}_R\mathfrak{M}$ , 219	
7 $\text{Ext}^1$ and Extensions, 226	
<b>List of Symbols</b>	<b>240</b>
<b>Bibliography</b>	<b>243</b>
<b>Index</b>	<b>245</b>