## Contents

Those sections that constitute the core of a modern algebra course are indicated by the symbol †

1. Introduction

## Classical Algebra, 1' Modern Algebra, 2 Binary Operations, 2 Algebraic Structures, 5 Extending Number Systems, 6 2. Boolean Algebras 8 Algebra of Sets, 8 Number of Elements in a Set, 13 Boolean Algebras, 15 Switching Circuits, 23 Posets and Lattices, 25 Normal Forms and Simplification of Circuits, 28 Transistor Gates, 39 Representation Theorem, 42 Exercises, 44

1

3.	Groups	51
	+Groups and Symmetries, 52 +Subgroups, 58 +Cyclic Groups and Dihedral Groups, 60 +Morphisms, 64 +Permutation Groups, 67 +Even and Odd Permutations, 73 Cayley's Representation Theorem, 77 Exercises, 78	
4.	<b>Quotient Groups</b>	82
	†Equivalence Relations, 82 †Cosets and Lagrange's Theorem, 85 †Normal Subgroups and Quotient Groups, 88 †Morphism Theorem, 93 †Direct Products, 98 Groups of Low Order, 102 Action of a Group on a Set, 104 Exercises, 108	
5.	Symmetry Groups in Three Dimensions	112
	Translations and the Euclidean Group, 112 Matrix Groups, 115 Finite Groups in Two Dimensions, 117 Proper Rotations of Regular Solids, 119 Finite Rotation Groups in Three Dimensions, 124 Crystallographic Groups, 129 Exercises, 130	
6.	Pólya-Burnside Method of Enumeration	
	1 orya-Dan asiac Michiga of Engineration	133
	Burnside's Theorem, 134 Necklace Problems, 135 Coloring Polyhedra, 138 Counting Switching Circuits, 140 Exercises, 145	133
7.	Burnside's Theorem, 134 Necklace Problems, 135 Coloring Polyhedra, 138 Counting Switching Circuits, 140	133

CONTENTS	•	٧i

	Quotient Monoids and the Monoid of a Machine, 155 Exercises, 160	
8.	Rings and Fields	166
	†Rings, 166 †Integral Domains and Fields, 171 †Subrings and Morphisms of Rings, 173 †New Rings from Old, 176 Field of Fractions, 182 Convolution Fractions, 184 Exercises, 188	
9.	Polynomial and Euclidean Rings	193
	†Division Algorithm, 193 †Euclidean Algorithm, 198 †Unique Factorization, 202 †Factoring Real and Complex Polynomials, 205 †Factoring Rational and Integral Polynomials, 207 †Factoring Polynomials over Finite Fields, 211 Linear Congruences and the Chinese Remainder Theorem, 212 Exercises, 217	
10.	Quotient Rings	221
	†Ideals and Quotient Rings, 221 †Computations in Quotient Rings, 225 †Morphism Theorem, 227 †Quotient Polynomial Rings that are Fields, 228 Exercises, 232	
11.	Field Extensions	235
	+Field Extensions, 235 +Algebraic Numbers, 238 +Galois Fields, 243 Primitive Elements, 246 Exercises, 252	
12.	Latin Squares	255
	Latin Squares, 255 Orthogonal Latin Squares, 258	

14.

CONTENTS
272
286

Constructible Numbers, 273 Duplicating the Cube, 278 Trisecting an Angle, 278
Squaring the Circle, 281
Constructing Regular Polygons, 281 A Nonconstructible Number of Degree Four, 283
Exercises, 284
Error-Correcting Codes
The Coding Problem, 287

Simple Codes, 290
Polynomial Representation, 293
Matrix Representation, 299
Error Correcting and Decoding, 304
BCH Codes, 309
Exercises, 314

Bibliography and References

Finite Geometries, 262 Magic Squares, 266 Exercises, 269

13. Geometrical Constructions

•	•	•	
Answers	to	the Odd-Numbered Exercises	

Glossary of Symbols

Index

337 341

319

324