

# List of Contents

Author's Preface . . . . .	9
<b>Chapter 1 GROUPS . . . . .</b>	
1.1 Introduction . . . . .	11
1.2 The Group Axioms and Basic Notions . . . . .	12
1.3 Examples of Groups . . . . .	14
1.4 The Study of Groups . . . . .	18
1.5 Free Groups, Generators and Relations . . . . .	19
1.6 Cosets, Normal Subgroups, and Lagrange's Theorem . . . . .	21
1.7 Conjugacy . . . . .	29
1.8 Factor Groups and Homomorphisms . . . . .	32
1.9 Permutation Representations and Permutation Groups . . . . .	38
1.10 The Alternating Groups . . . . .	43
1.11 The Sylow Theorems . . . . .	45
1.12 Automorphism Groups . . . . .	50
1.13 Direct and Semidirect Products . . . . .	51
1.14 Groups of Order $p^2$ . . . . .	54
1.15 The Construction of Finite Groups . . . . .	55
1.16 Exercises . . . . .	56
<b>Chapter 2 RINGS AND FIELDS . . . . .</b>	63
2.1 Introduction . . . . .	63
2.2 Axioms for Rings . . . . .	64
2.3 Ideals and Factor Rings . . . . .	66

2.4	Gaussian, Principal, and Euclidean Domains . . . . .	69
2.5	Dedekind Domains . . . . .	75
2.6	Noetherian Rings . . . . .	76
2.7	Fields . . . . .	81
2.8	The Nature of the Prime Fields II . . . . .	81
2.9	Extension Fields. . . . .	82
2.10	Simple Field Extensions . . . . .	84
2.11	Equivalence of Extensions . . . . .	85
2.12	Algebraic Field Extensions . . . . .	87
2.13	Splitting Fields . . . . .	89
2.14	Normal Extension Fields. . . . .	91
2.15	Roots of Unity . . . . .	93
2.16	Finite Fields . . . . .	96
2.17	The Construction of Finite Fields. . . . .	100
2.18	Error-Correcting Codes . . . . .	101
2.19	Separable and Inseparable Extensions . . . . .	105
2.20	Perfect and Imperfect Fields . . . . .	107
2.21	The Theorem of the Primitive Element . . . . .	108
2.22	The Galois Group . . . . .	110
2.23	Some Calculations . . . . .	114
2.24	Cyclic Fields . . . . .	116
2.25	Solvability of Equations by Radicals . . . . .	118
2.26	The Solvability by Radicals of the General Equation of Degree $n$ . . . . .	122
2.27	Geometrical Constructions . . . . .	124
2.28	Exercises . . . . .	128
 Chapter 3 LINEAR GROUPS . . . . .		131
3.1	Introduction . . . . .	131
3.2	The General Linear Group . . . . .	132
3.3	The Sylow Subgroups of $GL_n(K)$ . . . . .	134
3.4	Some Important Simple Groups . . . . .	135
3.5	Introduction to Matrix Representations of Groups . . . . .	144
3.6	Complete Reducibility . . . . .	153
3.7	Orthogonality Relations of the First Kind. . . . .	157
3.8	The Regular Representation . . . . .	163
3.9	Orthogonality Relations of the Second Kind. . . . .	166
3.10	The Character Table . . . . .	172
3.11	Representations of Factor Groups . . . . .	179
3.12	Representations from Subgroups . . . . .	184
3.13	Finding Normal Subgroups. . . . .	191
3.14	Some Theorems of Burnside . . . . .	191
3.15	Projection Operators and Symmetry Coordinates	192
3.16	Exercises . . . . .	195

<b>Chapter 4 COMPUTATIONAL GROUP THEORY . . . . .</b>	<b>201</b>
4.1 Introduction . . . . .	201
4.2 The Todd-Coxeter Algorithm . . . . .	201
4.3 Some Examples of Coset Enumeration . . . . .	210
4.4 Proof of Termination of Process for a Finite Index . . . . .	222
4.5 A Computer Implementation of the Todd-Coxeter Procedure . . . . .	224
4.6 Some Applications of Coset Enumeration . . . . .	229
4.7 Some Recent Developments. . . . .	237
4.8 Some Examples of Output from Todd-Coxeter Program . . . . .	242
4.9 Exercises . . . . .	261
<b>Outline Solutions to Selected Exercises . . . . .</b>	<b>263</b>
<b>Further Reading and References. . . . .</b>	<b>271</b>
<b>Index . . . . .</b>	<b>277</b>