

# Contents

Preface to the second edition	<i>page</i> xiii
Preface to the first edition	xiv
Glossary of symbols	xviii

## Part I Preliminaries

Chapter 1	Sets and mappings	3
	1. Sets	3
	2. Relations	9
	3. Mappings	14
	4. Binary operations	21
	5. Cardinality of a set	25
Chapter 2	Integers, real numbers, and complex numbers	30
	1. Integers	30
	2. Rational, real, and complex numbers	35
	3. Fields	36
Chapter 3	Matrices and determinants	39
	1. Matrices	39
	2. Operations on matrices	41
	3. Partitions of a matrix	46
	4. The determinant function	47
	5. Properties of the determinant function	49
	6. Expansion of $\det A$	53

## Part II Groups

Chapter 4	Groups	61
	1. Semigroups and groups	61
	2. Homomorphisms	69
	3. Subgroups and cosets	72
	4. Cyclic groups	82
	5. Permutation groups	84
	6. Generators and relations	90
Chapter 5	Normal subgroups	91
	1. Normal subgroups and quotient groups	91
	2. Isomorphism theorems	97
	3. Automorphisms	104
	4. Conjugacy and $G$ -sets	107
Chapter 6	Normal series	120
	1. Normal series	120
	2. Solvable groups	124
	3. Nilpotent groups	126
Chapter 7	Permutation groups	129
	1. Cyclic decomposition	129
	2. Alternating group $A_n$	132
	3. Simplicity of $A_n$	135
Chapter 8	Structure theorems of groups	138
	1. Direct products	138
	2. Finitely generated abelian groups	141
	3. Invariants of a finite abelian group	143
	4. Sylow theorems	146
	5. Groups of orders $p^2$ , $pq$	152

## Part III Rings and modules

Chapter 9	Rings	159
	1. Definition and examples	159
	2. Elementary properties of rings	161
	3. Types of rings	163
	4. Subrings and characteristic of a ring	168
	5. Additional examples of rings	176

Chapter 10	Ideals and homomorphisms	179
	1. Ideals	179
	2. Homomorphisms	187
	3. Sum and direct sum of ideals	196
	4. Maximal and prime ideals	203
	5. Nilpotent and nil ideals	209
	6. Zorn's lemma	210
Chapter 11	Unique factorization domains and euclidean domains	212
	1. Unique factorization domains	212
	2. Principal ideal domains	216
	3. Euclidean domains	217
	4. Polynomial rings over UFD	219
Chapter 12	Rings of fractions	224
	1. Rings of fractions	224
	2. Rings with Ore condition	228
Chapter 13	Integers	233
	1. Peano's axioms	233
	2. Integers	240
Chapter 14	Modules and vector spaces	246
	1. Definition and examples	246
	2. Submodules and direct sums	248
	3. $R$ -homomorphisms and quotient modules	253
	4. Completely reducible modules	260
	5. Free modules	263
	6. Representation of linear mappings	268
	7. Rank of a linear mapping	273

#### Part IV Field theory

Chapter 15	Algebraic extensions of fields	281
	1. Irreducible polynomials and Eisenstein criterion	281
	2. Adjunction of roots	285
	3. Algebraic extensions	289
	4. Algebraically closed fields	295

Chapter 16	Normal and separable extensions	300
	1. Splitting fields	300
	2. Normal extensions	304
	3. Multiple roots	307
	4. Finite fields	310
	5. Separable extensions	316
Chapter 17	Galois theory	322
	1. Automorphism groups and fixed fields	322
	2. Fundamental theorem of Galois theory	330
	3. Fundamental theorem of algebra	338
Chapter 18	Applications of Galois theory to classical problems	340
	1. Roots of unity and cyclotomic polynomials	340
	2. Cyclic extensions	344
	3. Polynomials solvable by radicals	348
	4. Symmetric functions	355
	5. Ruler and compass constructions	358
Part V Additional topics		
Chapter 19	Noetherian and artinian modules and rings	367
	1. $\text{Hom}_R(\oplus M_i, \oplus M_i)$	367
	2. Noetherian and artinian modules	368
	3. Wedderburn–Artin theorem	382
	4. Uniform modules, primary modules, and Noether–Lasker theorem	388
Chapter 20	Smith normal form over a PID and rank	392
	1. Preliminaries	392
	2. Row module, column module, and rank	393
	3. Smith normal form	394
Chapter 21	Finitely generated modules over a PID	402
	1. Decomposition theorem	402
	2. Uniqueness of the decomposition	404
	3. Application to finitely generated abelian groups	408
	4. Rational canonical form	409
	5. Generalized Jordan form over any field	418

Chapter 22	Tensor products	426
	1. Categories and functors	426
	2. Tensor products	428
	3. Module structure of tensor product	431
	4. Tensor product of homomorphisms	433
	5. Tensor product of algebras	436
	Solutions to odd-numbered problems	438
	Selected bibliography	476
	Index	477