

Table of Contents

| | |
|--|------------|
| Introduction | 1 |
| Part A. Basic Notions of Definability | 5 |
| <i>Chapter I. Groundwork</i> | <i>7</i> |
| 1. Logic and Set Theory | 7 |
| 2. Topology and Measure | 15 |
| 3. Inductive Definitions | 22 |
| <i>Chapter II. Ordinary Recursion Theory</i> | <i>27</i> |
| 1. Primitive Recursion | 28 |
| 2. Recursive Functionals and Relations | 37 |
| 3. Normal Forms | 46 |
| 4. Semi-Recursive Relations | 52 |
| 5. Relativization | 62 |
| <i>Chapter III. Hierarchies and Definability</i> | <i>69</i> |
| 1. The Arithmetical Hierarchy | 69 |
| 2. The Analytical Hierarchy | 80 |
| 3. Inductive Definability | 89 |
| 4. Implicit Definability and Bases | 106 |
| 5. Definability in Formal Languages for Arithmetic | 114 |
| 6. Arithmetical Forcing | 124 |
| Part B. The Analytical and Projective Hierarchies | 133 |
| <i>Chapter IV. The First Level</i> | <i>135</i> |
| 1. Π_1^1 and Well-Orderings | 135 |
| 2. The Boundedness Principle and Other Applications | 143 |

| | |
|--|-----|
| 3. The Borel Hierarchy | 156 |
| 4. The Effective Borel and Hyperarithmetical Hierarchies | 163 |
| 5. Cardinality, Measurability and Category | 180 |
| 6. Continuous Images | 188 |
| 7. Uniformization | 194 |
| | |
| <i>Chapter V.</i> Δ_2^1 and Beyond | 201 |
| 1. The Pre-Wellordering Property | 202 |
| 2. The Hypothesis of Constructibility | 214 |
| 3. The Hypothesis of Projective Determinacy | 221 |
| 4. Classical Hierarchies in Δ_1^1 | 236 |
| 5. Effective Hierarchies in Δ_1^1 | 246 |
| 6. A Hierarchy for Δ_2^1 | 251 |
| | |
| Part C. Generalized Recursion Theories | 257 |
| | |
| <i>Chapter VI.</i> Recursion in a Type-2 Functional | 259 |
| 1. Basic Properties | 259 |
| 2. Substitution Theorems | 271 |
| 3. Ordinal Comparison | 284 |
| 4. Relations Semi-Recursive in a Type-2 Functional | 291 |
| 5. Hierarchies of Relations Recursive in a Type-2 Functional | 307 |
| 6. Extended Functionals | 315 |
| 7. Recursive Type-3 Functionals and Relations | 335 |
| | |
| <i>Chapter VII.</i> Recursion in a Type-3 Functional | 343 |
| 1. Basic Properties | 343 |
| 2. Relations Semi-Recursive in a Type-3 Functional | 350 |
| 3. Hierarchies of Relations Recursive in a Type-3 Functional | 360 |
| 4. Higher Types | 363 |
| | |
| <i>Chapter VIII.</i> Recursion on Ordinals | 371 |
| 1. Recursive Ordinal Functions | 372 |
| 2. Recursively Regular Ordinals | 383 |
| 3. Ordinal Recursion and the Analytical Hierarchy | 393 |
| 4. Ordinal Recursion and Type-2 Functionals | 403 |
| 5. Stability | 412 |
| 6. Recursively Large Ordinals | 419 |
| 7. Ordinal Recursion and Constructible Sets | 430 |
| | |
| Epilogue | 445 |
| References | 459 |
| Global Notational Conventions | 467 |
| Special Notations | 469 |
| Index | 473 |