Contents

Preface xi

1 An Introduction to Matrices

	1.1	An Orientation 1	
	1.2	Basic Definitions 3	
	1.3	Equalities and Inequalities 7	
	1.4	Matrix Addition 9	
	1.5	Multiplication by a Scalar 11	
		Matrix Multiplication 13	
	1.7	Sigma Notation 19	
	1.8	History and Further Applications of Matrices	22
2	An	Introduction to Vectors 27	
	2.1	Basic Definitions 27	
	2.2	Geometric Representation of a Vector 29	
	2.3		
	2.4	History and Further Applications of Vectors	40

viii Contents

3	Alg	ebraic Systems 43
	3.1	Definitions 43
	3.2	Matrix Operations and Their Properties 48
	3.3	<u>-</u>
	3.4	The Identity and the Inverse Elements 56
	3.5	Field—An Algebraic System (Optional) 59
	3.6	Ring—An Algebraic System (Optional) 61
	3.7	Group—An Algebraic System (Optional) 64
	3.8	Summary 65
4	Spe	ecial Matrices 67
	4.1	Transpose of a Matrix 67
	4.2	Symmetric Matrix 70
	4.3	Skew-Symmetric Matrix 72
	4.4	Hermitian Matrix (Optional) 73
	4.5	Triangular and Echelon Matrices 76
	4.6	Diagonal and Scalar Matrices 78
	4.7 4.8	Submatrices and Partitioning 80 Coefficient and Augmented Matrices 83
	4.9	Summary 84
5	Det	terminants 86
	5.1	Definition of a Determinant 86
	5.2	- · · · · · · · · · · · · · · · · · · ·
	5.3	Properties of Determinants 94
	5.4	Proofs of Determinant Theorems 1 through 10 (Optional) 100
	5.5	The Determinant of a Triangular Matrix 104
	5.6	Rank of a Matrix 106
	5.7	Cramer's Rule (Optional) 108
6	The	Inverse Matrix 111
	6.1	Introduction 111
	6.2	The Cofactor Matrix and Adjoint Matrix 112
	6.3	Inverse Matrix 113
	6.4	Properties of an Inverse Matrix 118
	6.5	Solution of n Linear Equations with n Unknowns 121
	6.6	Proof of Cramer's Rule (Optional) 124

6.7 Summary

125

ix

Conte	ents			
7	Elementary Matrix Transformations 126			
	7.1	Elementary Row Operations 126		
	7.2	Normal Form and Rank 129		
	7.3	Elementary Matrices 134		
	7.4	Calculation of the Inverse by Elementary Operations 137		
8	Sys	stems of Linear Equations 141		
	8.1	Introduction 141		
	8.2	Inconsistent Systems 147		
	8.3			
	8.4	Consistent System with an Infinite Number of Solutions 154		
	8.5	• • • • •		
	8.6	Summary 162		
9	Co	nvex Sets 164		
	9.1	Linear Inequalities 164		
	9.2	Convex Sets 170		

Maximum and Minimum of a Linear Function over a

182

205

213 218

10.9 Simultaneous Solutions of Primal and Dual Problems

182

195

192

201

227

222

175

Polyhedral Convex Set

10.1 A Geometric Method of Solution 10.2 History and Applications

10.4 An Algebraic Method of Solution

10.3 Matrix Notation and the General Case

10.5 An Introduction to the Simplex Method

10.10 More about the Primal-Dual Approach

234

231

Linear Programming

10.6 The Simplex Method

10.7 Artificial Variables

10.8 The Dual Problem

10.11 Degeneracy

10.12 Summary

9.3

10

x Contents

	11.1	Definition 236		
	11.2	A Spanning Set of Vectors 240		
	11.3	Linear Dependence and Independence 244		
	11.4	Basis of a Vector Space 247		
	11.5	Linear Programming and Bases (Optional) 251		
12	Linear Transformations and the Characteristic Value Problem 255			
	12.1	A Geometric Discussion of a Transformation 255		
	12.2	Matrix Transformations 259		
	12.3	Linear Transformations 262		
	10.4	Characteristic Values and Vectors of a Matrix 265		
	12.4	Characteristic values and vectors of a Matrix 200		

Index 309