## **Contents**

(Sections marked † are optional.)

	•	
	PREFACE	ix
	CHAPTER 1 · CONCRETE VECTOR SPACES	1
1.1	Vectors in the Plane 1	
1.2	The Dimension of $\Re^2$ 6	
1.3	Real and Complex n-Space 14	
	Review Quiz 22	
	CHAPTER 2 · ABSTRACT VECTOR SPACES	25
2.1	Definitions and Preliminary Theorems 25	
2.2	Linear Independence in a Vector Space 33	
<sup>†</sup> 2.3	Linear Independence of Functions 37	
2.4	Dimension of a Vector Space 42	
2.5	The Minimax Principle 50	
2.6	Subspaces of a Vector Space 53	
	Review Quiz 60	
	CHAPTER 3 · LINEAR MAPS	63
3.1	Definitions and Examples 63	
3.2	The Law of Nullity 74	
3.3	Invertible Linear Maps 83	
3.4	Concrete Spaces Revisited 91	
3.5	The Vector Space of Linear Maps 97	
	Review Quiz 104	

	CHAPTER 4 · MATRICES	107
4.1	The Matrix Representation of a Linear Map 107	
4.2	·	
4.3		
	Review Quiz 128	
	CHAPTER 5 · MULTIPLICATION OF MAPS AND MATRICES	131
5.1	Composition of Linear Maps 131	
5.2	The Inverse of a Linear Map 136	
5.3	Multiplication of Matrices 142	
5.4	The Inverse of a Matrix 153	
	Review Quiz 164	
	CHAPTER 6 · EQUIVALENCE AND SIMILARITY	167
6.1	Matrix Representations Revisited 167	
6.2		
6.3	Change of Basis 178	
	Review Quiz 188	
	CHAPTER 7 · LINEAR SYSTEMS	191
7.1	Systems of Linear Equations 191	
7.2	Row Echelon Form 199	
7.3	Row Operations 207	
7.4	Column Operations 219	
7.5	Rank 223	
7.6	Theory of Linear Systems 231	
	Review Quiz 235	
	CHAPTER 8 · DETERMINANTS	239
8.1	Definitions 239	
8.2	Expansion by a Row 244	
8.3	Expansion by a Column 252	
8.4	Uniqueness of Determinants 254	
<sup>†</sup> 8.5	Cramer's Rule 257	
	Review Quiz 264	

Contents	/	vii

	<b>4</b>	
	CHAPTER 9 · EIGENVALUES	267
9.1	Diagonal Matrices 267	
9.2	Eigenvalues of a Linear Operator 271	
9.3	Eigenvalues of a Matrix 278	
9.4	Diagonalization of a Matrix 285	
†9.5	An Application to Differential Equations 295	
†9.6	The Cayley-Hamilton Theorem 297	
	Review Quiz 300	
	CHAPTER 10 · INNER PRODUCTS	303
10.1	Geometry in n-Space 304	
10.1	Euclidean and Unitary Spaces 313	
10.3	The Cauchy-Schwarz Inequality 320	
10.4	Orthogonality 324	
10.5	Parseval's Identity 334	
<sup>†</sup> 10.6	Gram-Schmidt Orthogonalization 345	
	Review Quiz 352	
	CHAPTER 11 · THE PROJECTION THEOREM	355
		555
11.1	Sums of Subspaces 355	
11.2	Orthogonal Subspaces of a Unitary Space 362	
11.3	The Projection Theorem 369	
†11.4	Applications to Geometry and Analysis 375	
	Review Quiz 384	
	CHAPTER 12 · LINEAR OPERATORS ON A UNITARY SPACE	387
12.1	The Adjoint of a Linear Operator 387	
12.2	Orthogonal and Unitary Operators 396	
<sup>†</sup> 12.3	Orthogonal Transformations of the Plane 406	
†12.4	An Example in Hilbert Space 410	
12.5	Another Characterization of Unitary Operators 413	
12.6	Self-adjoint Operators 417	
	Review Quiz 425	

## viii / Contents

	CHAPTER 13 · SPECTRAL THEORY		429
13.1 †13.2	- Paris Maraina for Generalons	429	
13.3	Normal Operators and the Spectral Theorem Review Quiz 451	443	
	APPENDIX 1		453
A1.1	The Dual Space of a Vector Space 453		
A1.2	The Representation Theorem 455		
A1.3	Orthogonality Revisited 459		
	APPENDIX 2		464
A2.1	Triangulation of a Matrix 464		
A2.2			
	BIBLIOGRAPHY		474
	SELECTED ANSWERS AND HINTS		476
	INDEX		518