

CONTENTS

CHAPTER 1. INTRODUCTION	1
1-1 Linear models	1
1-2 Linear algebra	2
1-3 Leontief's interindustry model of an economy	3
1-4 Linear programming	6
1-5 Graphical solution of a linear programming problem in two variables	8
1-6 Regression analysis	10
1-7 Linear circuit theory	12
1-8 Other linear models	14
1-9 The road ahead	14
 CHAPTER 2. VECTORS	17
2-1 Physical motivation for the vector concept	17
2-2 Operations with vectors	19
2-3 The scalar product	22
2-4 Generalization to higher dimensions	25
2-5 Generalized vector operations	28
2-6 Euclidean space and the scalar product	30
2-7 Linear dependence	34
2-8 The concept of a basis	39
2-9 Changing a single vector in a basis	41
2-10 Number of vectors in a basis for E^n	43
2-11 Orthogonal bases	45
2-12 Generalized coordinate systems	48
2-13 Vector spaces and subspaces	50
 CHAPTER 3. MATRICES AND DETERMINANTS	60
3-1 Matrices	60
3-2 Matrix operations	61
3-3 Matrix multiplication—introduction	64
3-4 Matrix multiplication—further development	68
3-5 Vectors and matrices	71
3-6 Identity, scalar, diagonal, and null matrices	73
3-7 The transpose	76
3-8 Symmetric and skew-symmetric matrices	78
3-9 Partitioning of matrices	79
3-10 Basic notion of a determinant	85
3-11 General definition of a determinant	85
3-12 Some properties of determinants	88

3-13 Expansion by cofactors	90
3-14 Additional properties of determinants	93
3-15 Laplace expansion	95
3-16 Multiplication of determinants	99
3-17 Determinant of the product of rectangular matrices	100
3-18 The matrix inverse	103
3-19 Properties of the inverse	105
3-20 Computation of the inverse by partitioning	107
3-21 Product form of the inverse	111
3-22 Matrix series and the Leontief inverse	116
CHAPTER 4. LINEAR TRANSFORMATIONS, RANK, AND ELEMENTARY TRANSFORMATIONS	132
4-1 Definition of linear transformations	132
4-2 Properties of linear transformations	135
4-3 Rank	138
4-4 Rank and determinants	140
4-5 Elementary transformations	144
4-6 Echelon matrices and rank	148
CHAPTER 5. SIMULTANEOUS LINEAR EQUATIONS	162
5-1 Introduction	162
5-2 Gaussian elimination	162
5-3 Cramer's rule	166
5-4 Rules of rank	167
5-5 Further properties	170
5-6 Homogeneous linear equations	173
5-7 Geometric interpretation	177
5-8 Basic solutions	178
CHAPTER 6. CONVEX SETS AND n-DIMENSIONAL GEOMETRY	188
6-1 Sets	188
6-2 Point sets	189
6-3 Lines and hyperplanes	194
6-4 Convex sets	202
6-5 The convex hull	207
6-6 Theorems on separating hyperplanes	210
6-7 A basic result in linear programming	215
6-8 Convex hull of extreme points	217
6-9 Introduction to convex cones	219
6-10 Convex polyhedral cones	224
6-11 Linear transformations of regions	227

CHAPTER 7. CHARACTERISTIC VALUE PROBLEMS AND QUADRATIC FORMS	236
7-1 Characteristic value problems	236
7-2 Similarity	238
7-3 Characteristic value problems for symmetric matrices	239
7-4 Additional properties of the eigenvectors of a symmetric matrix	242
7-5 Diagonalization of symmetric matrices	247
7-6 Characteristic value problems for nonsymmetric matrices	249
7-7 Quadratic forms	251
7-8 Change of variables	253
7-9 Definite quadratic forms	254
7-10 Diagonalization of quadratic forms	255
7-11 Diagonalization by completion of the square	257
7-12 Another set of necessary and sufficient conditions for positive and negative definite forms	259
7-13 Simultaneous diagonalization of two quadratic forms	263
7-14 Geometric interpretation; coordinates and bases	264
7-15 Equivalence and similarity	267
7-16 Rotation of coordinates; orthogonal transformations	269