

Contents

Preface ix

1 Background in abstract algebra	1
1.0 Introduction	1
1.1 Sets, elements, basic notations	1
1.2 Mappings	4
1.3 Multiplication of mappings	6
1.4 Relations; equivalence relations	11
1.5 Binary operations	14
1.6 Groups	18
1.7 Additive notations	22
1.8 Homomorphisms and isomorphisms	24
1.9 Rings and integral domains	28
1.10 Fields	32
1.11 Ordered integral domains	34
1.12 Integers and induction	38
1.13 The integers modulo m	49
1.14 Finite and infinite sets	51
1.15 Rational, real, and complex numbers	56
1.16 Polynomials	59
2 Vector spaces	65
2.1 Definition of vector space	65
2.2 Subspaces	74
2.3 The subspace spanned by a set of vectors	80
2.4 Linear dependence and independence	87
2.5 A basic theorem	94
2.6 Basis and dimension	96
2.7 Cosets and quotient spaces	107
2.8 Direct sums of subspaces	112

3	Some geometry in vector spaces	118
3.0	Introduction	118
3.1	Points and arrows	118
3.2	Lines and line segments	125
3.3	Translations	138
3.4	Planes and flats	141
3.5	The flat determined by a set of points	153
3.6	Barycentric combinations and coordinates	160
3.7	The dot product; real Euclidean spaces	173
3.8	Orthogonal sets	183
3.9	Cosines, sines, and angles	198
3.10	Orthogonal coordinate systems in \mathbf{R}_n	204
4	Linear transformations and matrices	210
4.0	Introduction	210
4.1	Linear transformations	210
4.2	L.T.'s, bases, and matrices	216
4.3	Algebraic operations on L.T.'s	228
4.4	Algebraic operations on matrices	231
4.5	Miscellaneous results	239
4.6	The range and nullspace of a linear transformation	245
4.7	L.T.'s which are one-to-one or onto; left and right inverses	250
4.8	Change of basis	255
4.9	Affine transformations	261
4.10	Isometries and congruence	272
5	Matrices and determinants	287
5.0	Introduction	287
5.1	Permutations	287
5.2	Definition of the determinant	295
5.3	The expansion by minors	299
5.4	Properties of determinants	303
5.5	Row and column operations on matrices	309
5.6	Echelon form of a matrix	313
5.7	Elementary matrices, areas, and volumes	323
5.8	The computation of A^{-1}	331
5.9	The rank of a matrix	334
5.10	Systems of linear equations	337
5.11	Eigenvectors and eigenvalues	348
5.12	The coefficients of the characteristic polynomial	354
5.13	Some special cases	359
5.14	The generalized cross product	364

6 Further study of isometries	370
6.1 Rotations in \mathbf{R}_2	370
6.2 Oriented angles in \mathbf{R}_2	377
6.3 The isometries of \mathbf{R}_2	391
6.4 Rotations in higher dimensions	403
6.5 The isometries of \mathbf{R}_3	409
6.6 Rigid motions and rigid mappings	418
6.7 Orientation	422
7 Conics and quadrics in \mathbf{R}_n	429
7.0 Introduction	429
7.1 Equations of the second degree	430
7.2 The rank of the extended matrix	444
7.3 Classification of second-degree equations in \mathbf{R}_2	447
7.4 Quadric surfaces in \mathbf{R}_3	454
7.5 Conic sections	459
7.6 Singular points and the rank of M^*	463
7.7 An interpretation of projective spaces within vector spaces	476
8 The structure of linear transformations	488
8.0 Introduction	488
8.1 Further theorems about the range and nullspace	489
8.2 Nilpotent transformations	491
8.3 The (\mathbf{T}, λ) -characteristic subspace	502
8.4 The Jordan form	506
8.5 The Cayley-Hamilton theorem	519
9 The vector space \mathbf{C}_n and related matters	522
9.0 Introduction	522
9.1 Some basic definitions in \mathbf{C}_n	523
9.2 Real linear transformations of \mathbf{C}_n	527
9.3 Isometries of \mathbf{C}_n	530
9.4 The isometries of \mathbf{R}_n	532
9.5 Eigenvectors of Hermitian and real symmetric matrices	540
9.6 Symmetric and Hermitian forms	543
9.7 Positive definite matrices	545
9.8 Maxima and minima of real functions of n variables	550
Appendixes	555
A Proof of Theorem 4.10.4	555
B The cycle representation of permutations	558
C Proof of Theorem 5.7.4	560
D Proof of Theorem 4.9.9	562

E	Some applications to analysis	565
F	Equations of selected conics and 3-quadrics	572
G	Hints and answers to selected exercises	581
	References	614
	Index of notation	615
	Index	619