

Contents

<i>Preface</i>	xiii
Introduction	1
Part I MONIC MATRIX POLYNOMIALS	9
Chapter 1 Linearization and Standard Pairs	
1.1 Linearization	11
1.2 Application to Differential and Difference Equations	15
1.3 The Inverse Problem for Linearization	20
1.4 Jordan Chains and Solutions of Differential Equations	23
1.5 Root Polynomials	29
1.6 Canonical Set of Jordan Chains	32
1.7 Jordan Chains and the Singular Part of the Laurent Expansion	37
1.8 Definition of a Jordan Pair of a Monic Polynomial	40
1.9 Properties of a Jordan Pair	43
1.10 Standard Pairs of a Monic Matrix Polynomial	46
Chapter 2 Representation of Monic Matrix Polynomials	
2.1 Standard and Jordan Triples	50
2.2 Representations of a Monic Matrix Polynomial	57
2.3 Resolvent Form and Linear Systems Theory	66
2.4 Initial Value Problems and Two-Point Boundary Value Problems	70
2.5 Complete Pairs and Second-Order Differential Equations	75
2.6 Initial Value Problem for Difference Equations, and the Generalized Newton Identities	79

Chapter 3 Multiplication and Divisibility

3.1	A Multiplication Theorem	85
3.2	Division Process	89
3.3	Characterization of Divisors and Supporting Subspaces	96
3.4	Example	100
3.5	Description of the Quotient and Left Divisors	104
3.6	Divisors and Supporting Subspaces for the Adjoint Polynomial	111
3.7	Decomposition into a Product of Linear Factors	112

Chapter 4 Spectral Divisors and Canonical Factorization

4.1	Spectral Divisors	116
4.2	Linear Divisors and Matrix Equations	125
4.3	Stable and Exponentially Growing Solutions of Differential Equations	129
4.4	Left and Right Spectral Divisors	131
4.5	Canonical Factorization	133
4.6	Theorems on Two-Sided Canonical Factorization	139
4.7	Wiener–Hopf Factorization for Matrix Polynomials	142

Chapter 5 Perturbation and Stability of Divisors

5.1	The Continuous Dependence of Supporting Subspaces and Divisors	147
5.2	Spectral Divisors: Continuous and Analytic Dependence	150
5.3	Stable Factorizations	152
5.4	Global Analytic Perturbations: Preliminaries	155
5.5	Polynomial Dependence	158
5.6	Analytic Divisors	162
5.7	Isolated and Nonisolated Divisors	164

Chapter 6 Extension Problems

6.1	Statement of the Problems and Examples	167
6.2	Extensions via Left Inverses	169
6.3	Special Extensions	173

Part II NONMONIC MATRIX POLYNOMIALS 181**Chapter 7 Spectral Properties and Representations**

7.1	The Spectral Data (Finite and Infinite)	183
7.2	Linearizations	186
7.3	Decomposable Pairs	188
7.4	Properties of Decomposable Pairs	191
7.5	Decomposable Linearization and a Resolvent Form	195

7.6	Representation and the Inverse Problem	197
7.7	Divisibility of Matrix Polynomials	201
7.8	Representation Theorems for Comonic Matrix Polynomials	206
7.9	Comonic Polynomials from Finite Spectral Data	208
7.10	Description of Divisors via Invariant Subspaces	211
7.11	Construction of a Comonic Matrix Polynomial via a Special Generalized Inverse	213
Chapter 8 Applications to Differential and Difference Equations		
8.1	Differential Equations in the Nonmonic Case	219
8.2	Difference Equations in the Nonmonic Case	225
8.3	Construction of Differential and Difference Equations with Given Solutions	227
Chapter 9 Least Common Multiples and Greatest Common Divisors of Matrix Polynomials		
9.1	Common Extensions of Admissible Pairs	232
9.2	Common Restrictions of Admissible Pairs	235
9.3	Construction of l.c.m. and g.c.d. via Spectral Data	239
9.4	Vandermonde Matrix and Least Common Multiples	240
9.5	Common Multiples for Monic Polynomials	244
9.6	Resultant Matrices and Greatest Common Divisors	246
Part III SELF-ADJOINT MATRIX POLYNOMIALS		253
Chapter 10 General Theory		
10.1	Simplest Properties	255
10.2	Self-Adjoint Triples: Definition	260
10.3	Self-Adjoint Triples: Existence	263
10.4	Self-Adjoint Triples for Real Self-Adjoint Matrix Polynomials	266
10.5	Sign Characteristic of a Self-Adjoint Matrix Polynomial	273
10.6	Numerical Range and Eigenvalues	276
Chapter 11 Factorization of Self-Adjoint Matrix Polynomials		
11.1	Symmetric Factorization	278
11.2	Main Theorem	279
11.3	Proof of the Main Theorem	282
11.4	Discussion and Further Deductions	285
Chapter 12 Further Analysis of the Sign Characteristic		
12.1	Localization of the Sign Characteristic	290
12.2	Stability of the Sign Characteristic	293

12.3	A Sign Characteristic for Self-Adjoint Analytic Matrix Functions	294
12.4	Third Description of the Sign Characteristic	298
12.5	Nonnegative Matrix Polynomials	301

Chapter 13 Quadratic Self-Adjoint Polynomials

13.1	Overdamped Case	305
13.2	Weakly Damped Case	309

Part IV SUPPLEMENTARY CHAPTERS IN LINEAR ALGEBRA

311

Chapter S1 The Smith Form and Related Problems

S1.1	The Smith Form	313
S1.2	Invariant Polynomials and Elementary Divisors	319
S1.3	Application to Differential Equations with Constant Coefficients	321
S1.4	Application to Difference Equations	325
S1.5	Local Smith Form and Partial Multiplicities	330
S1.6	Equivalence of Matrix Polynomials	333
S1.7	Jordan Normal Form	336
S1.8	Functions of Matrices	337

Chapter S2 The Matrix Equation $AX - XB = C$

S2.1	Existence of Solutions of $AX - XB = C$	342
S2.2	Commuting Matrices	345

Chapter S3 One-Sided and Generalized Inverses

348

Chapter S4 Stable Invariant Subspaces

S4.1	Projectors and Subspaces	353
S4.2	Spectral Invariant Subspaces and Riesz Projectors	356
S4.3	The Gap between Subspaces	360
S4.4	The Metric Space of Subspaces	363
S4.5	Stable Invariant Subspaces: Definition and Main Result	366
S4.6	Case of a Single Eigenvalue	367
S4.7	General Case of Stable Invariant Subspaces	369

Chapter S5 Indefinite Scalar Product Spaces

S5.1	Canonical Form of a Self-Adjoint Matrix and the Indefinite Scalar Product	376
S5.2	Proof of Theorem S5.1	378

CONTENTS	xi
S5.3 Uniqueness of the Sign Characteristic	383
S5.4 Second Description of the Sign Characteristic	386
<i>Chapter S6 Analytic Matrix Functions</i>	
S6.1 General Results	388
S6.2 Analytic Perturbations of Self-Adjoint Matrices	394
References	397
<i>List of Notations and Conventions</i>	403
<i>Index</i>	405