

# T A B L E O F C O N T E N T S

INTRODUCTION	. . . . .	1
PART I.	BASIC THEORY . . . . .	9
CHAPTER 1.	INDEFINITE SCALAR PRODUCTS	
1.1	Definition . . . . .	10
1.2	Orthogonality and orthogonal bases . . . . .	12
1.3	Classification of subspaces . . . . .	15
CHAPTER 2.	CLASSES OF LINEAR TRANSFORMATIONS	
2.1	Adjoint matrices . . . . .	19
2.2	H-selfadjoint matrices; examples and simplest properties . . . . .	22
2.3	H-unitary matrices; examples and simplest properties . . . . .	25
2.4	A second characterization of H-unitary matrices . . . . .	29
CHAPTER 3.	CANONICAL FORMS OF H-SELFADJOINT MATRICES	
3.1	Unitary similarity . . . . .	31
3.2	Description of a canonical form . . . . .	33
3.3	First applications of the canonical form . . . . .	35
3.4	Proof of Theorem 3.3 . . . . .	37
3.5	Classification of matrices by unitary similarity . . . . .	43
3.6	Signature matrices . . . . .	47
3.7	The structure of H-selfadjoint matrices when H has a small number of negative eigenvalues . . . . .	52

3.8	H-definite matrices . . . . .	54
3.9	Second description of the sign characteristic . . . . .	55
3.10	Canonical forms for pairs of hermitian matrices . . . . .	57
3.11	Third description of the sign characteristic . . . . .	59
3.12	Maximal nonnegative invariant subspaces . . . . .	61
3.13	Inverse problems . . . . .	68
 <b>CHAPTER 4. CANONICAL FORMS FOR H-UNITARY MATRICES</b>		
4.1	First examples of canonical forms . . .	70
4.2	Canonical forms in the general case . . .	73
4.3	Correctness of the sign characteristic. .	79
4.4	First applications of the canonical form . . . . .	82
4.5	Further deductions from the canonical form . . . . .	83
4.6	H-normal matrices . . . . .	84
 <b>CHAPTER 5. REAL MATRICES</b>		
5.1	Real H-selfadjoint matrices and canonical forms . . . . .	87
5.2	Proof of Theorem 5.3 . . . . .	91
5.3	Comparison with results in the complex case . . . . .	95
5.4	Connected components of real unitary similarity classes . . . . .	96
5.5	Connected components of real unitary similarity classes (H fixed) . . . . .	102
 <b>CHAPTER 6. FUNCTIONS OF H-SELFADJOINT AND H-UNITARY MATRICES</b>		
6.1	Preliminaries . . . . .	106
6.2	Exponential and logarithmic functions . .	108

6.3	Functions of H-selfadjoint matrices . . .	110
6.4	Functions of H-unitary matrices . . . .	114
6.5	The canonical form and sign characteristic for a function of an H-selfadjoint matrix . . . . .	115
6.6	Functions of H-selfadjoint matrices which are selfadjoint in another indefinite scalar product . . . . .	119
NOTES TO PART I . . . . .		121
PART II. FIRST APPLICATIONS . . . . .		123
CHAPTER 1. HAMILTONIAN AND SELFADJOINT DIFFERENTIAL EQUATIONS WITH PERIODIC COEFFICIENTS		
1.1	The matrizant . . . . .	126
1.2	The monodromy matrix . . . . .	131
1.3	The Floquet theorem . . . . .	133
1.4	The real case . . . . .	134
1.5	Proof of Theorem 1.7 . . . . .	136
1.6	Selfadjoint equations with periodic coefficients . . . . .	138
1.7	The real case for selfadjoint equations . . . . .	144
1.8	Boundedness of solutions of self- adjoint equations . . . . .	145
CHAPTER 2. HERMITIAN MATRIX POLYNOMIALS		
2.1	Preliminaries . . . . .	149
2.2	Matrix polynomials with hermitian coefficients . . . . .	152
2.3	Factorization of hermitian matrix polynomials . . . . .	156
2.4	Difference equations and hermitian matrix polynomials on the unit circle . .	160

<b>CHAPTER 3. HERMITIAN RATIONAL MATRIX FUNCTIONS</b>	
3.1	Minimal nodes . . . . . 166
3.2	The sign characteristic: definition and main result . . . . . 168
3.3	Null functions and Jordan chains . . . . . 171
3.4	Proof of Theorem 3.4 . . . . . 174
3.5	Factorization of hermitian rational matrix functions . . . . . 177
3.6	Symmetric factorizations of hermitian rational matrix functions . . . . . 179
3.7	Nonnegative definite rational matrix functions . . . . . 183
3.8	Minimal factorizations of real hermitian rational matrix functions . . . . . 186
<b>APPENDIX TO CHAPTER 3. RATIONAL MATRIX FUNCTIONS</b>	
A.1	Linear systems, their transfer functions and nodes . . . . . 188
A.2	The local Smith form and partial multiplicities . . . . . 193
A.3	Minimal factorizations . . . . . 195
A.4	Minimal factorizations of real rational functions . . . . . 199
<b>CHAPTER 4. THE ALGEBRAIC RICCATI EQUATION</b>	
4.1	An optimal control problem . . . . . 202
4.2	General solutions of the Riccati equation . . . . . 203
4.3	Existence of hermitian solutions of the Riccati equation . . . . . 208
4.4	Hermitian solutions and nonnegative rational functions . . . . . 214
4.5	Description of hermitian solutions . . . . . 217
4.6	Extremal hermitian solutions . . . . . 219
4.7	Real symmetric solutions of the algebraic Riccati equation with real coefficients . . . . . 223

NOTES TO PART II . . . . .	226
PART III. PERTURBATIONS AND STABILITY . . . . .	227
CHAPTER 1. GENERAL PERTURBATIONS. STABILITY OF DIAGONABLE MATRICES	
1.1 General perturbations of H-selfadjoint matrices . . . . .	229
1.2 Stably diagonalizable H-selfadjoint matrices . . . . .	232
1.3 General perturbations and stably diagonalizable H-unitary matrices . . . . .	235
1.4 Analytic perturbations and eigenvalues . .	236
1.5 Analytic perturbations and eigenvectors . . . . .	241
1.6 The real case . . . . .	243
1.7 The real skew-symmetric case . . . . .	244
1.8 Auxiliary results for the real skew- symmetric case . . . . .	246
1.9 Proof of Theorem 1.10 . . . . .	248
CHAPTER 2. APPLICATIONS TO DIFFERENTIAL AND DIFFERENCE EQUATIONS	
2.1 Differential equations of first order . .	251
2.2 Differential equations of higher order . .	252
2.3 The strongly hyperbolic case . . . . .	255
2.4 Difference equations . . . . .	257
2.5 Hamiltonian and selfadjoint equations . .	260
CHAPTER 3. POSITIVE PERTURBATIONS	
3.1 Positive perturbations of H-selfadjoint matrices . . . . .	262
3.2 Hamiltonian systems of positive type with constant coefficients . . . . .	265

<b>CHAPTER 4.</b>	<b>PERTURBATIONS OF INVARIANT MAXIMAL NEUTRAL SUBSPACES</b>	
4.1	Continuity of invariant maximal neutral subspaces . . . . .	268
4.2	Analyticity of invariant maximal neutral subspaces . . . . .	270
4.3	Extremal solutions of the algebraic Riccati equation . . . . .	273
4.4	Application to the optimal control problem . . . . .	275
4.5	Continuity of canonical factorization of nonnegative rational matrix functions . . . . .	278
<b>CHAPTER 5.</b>	<b>PERTURBATIONS WHICH PRESERVE JORDAN STRUCTURE</b>	
5.1	Stability of the sign characteristic . . . . .	282
5.2	Stability of unitary similarity . . . . .	287
5.3	Special cases of unitary similarity . . . . .	289
5.4	Continuous dependence of the canonical form . . . . .	291
5.5	Analytic dependence of the canonical form . . . . .	297
5.6	H-unitary matrices . . . . .	298
5.7	Connected components of selfadjoint matrices with like real Jordan structure . . . . .	299
5.8	The real case . . . . .	303
<b>APPENDIX TO PART III.</b>	<b>SUBSPACES IN FINITE DIMENSIONAL COMPLEX SPACE</b>	
A.1	The metric space of subspaces . . . . .	306
A.2	Continuous families of subspaces . . . . .	311
A.3	Analytic families of subspaces . . . . .	315
<b>NOTES TO PART III</b>		<b>318</b>

PART IV. CONNECTED COMPONENTS OF DIFFERENTIAL EQUATIONS. 319

CHAPTER 1. CONNECTED COMPONENTS OF STABLY DIAGONABLE MATRICES

1.1	H-selfadjoint stably r-diagonable matrices . . . . .	320
1.2	H-unitary stably u-diagonable matrices . . . . .	323
1.3	E-orthogonal stably u-diagonable matrices . . . . .	326

CHAPTER 2. DIFFERENTIAL AND DIFFERENCE EQUATIONS WITH CONSTANT COEFFICIENTS

2.1	Connected components of differential equations with hermitian coefficients and stably bounded solutions . . . . .	329
2.2	A special case . . . . .	331
2.3	Connected components of difference equations . . . . .	334

CHAPTER 3. CONNECTED COMPONENTS OF HAMILTONIAN EQUATIONS

3.1	Definition and explanation of the problem . . . . .	338
3.2	Group of H-unitary matrices . . . . .	340
3.3	Simple connectedness of unitary similarity classes . . . . .	343
3.4	Homotopy in connected components of u-stably diagonalizable matrices . . . . .	347
3.5	Homotopy indices of H-unitary matrices . . . . .	350
3.6	Connected components of hamiltonian systems with stably bounded solutions . . . . .	354
3.7	Simple connectedness of connected components of real E-orthogonal matrices . . . . .	356
3.8	Connected components of real hamiltonian systems with stably bounded solutions . . . . .	360

NOTES TO PART IV . . . . .	362
REFERENCES . . . . .	363
LIST OF NOTATIONS . . . . .	368
INDEX . . . . .	370